STiCM

Select / Special Topics in Classical Mechanics

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STiCM Lecture 15:

Unit 5 : Non-Inertial Frame of Reference

Real effects of pseudo-forces!

Unit 5: Inertial and non-inertial reference frames.

Moving coordinate systems. Pseudo forces.
Inertial and non-inertial reference frames.

Deterministic cause-effect relations in inertial frame, and their *modifications* in a non-inertial frame.

Real Effects of Pseudo Forces!

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Six Flags over Georgia

Gaspard Gustave de Coriolis
1792 - 1843

Learning goals:

Understand “Newton’s laws hold only in an inertial frame”.

Distinction between an inertial and a non-inertial frame is linked to what we consider as a fundamental physical force/interaction.

Electromagnetic/electroweak, nuclear, gravity) / pseudo-force (centrifugal, Coriolis etc.) / Friction

We shall learn to interpret the ‘real effects of pseudo-forces’ in terms of Newtonian method. *Re-activate* ‘causality’ in non-inertial frame of reference!

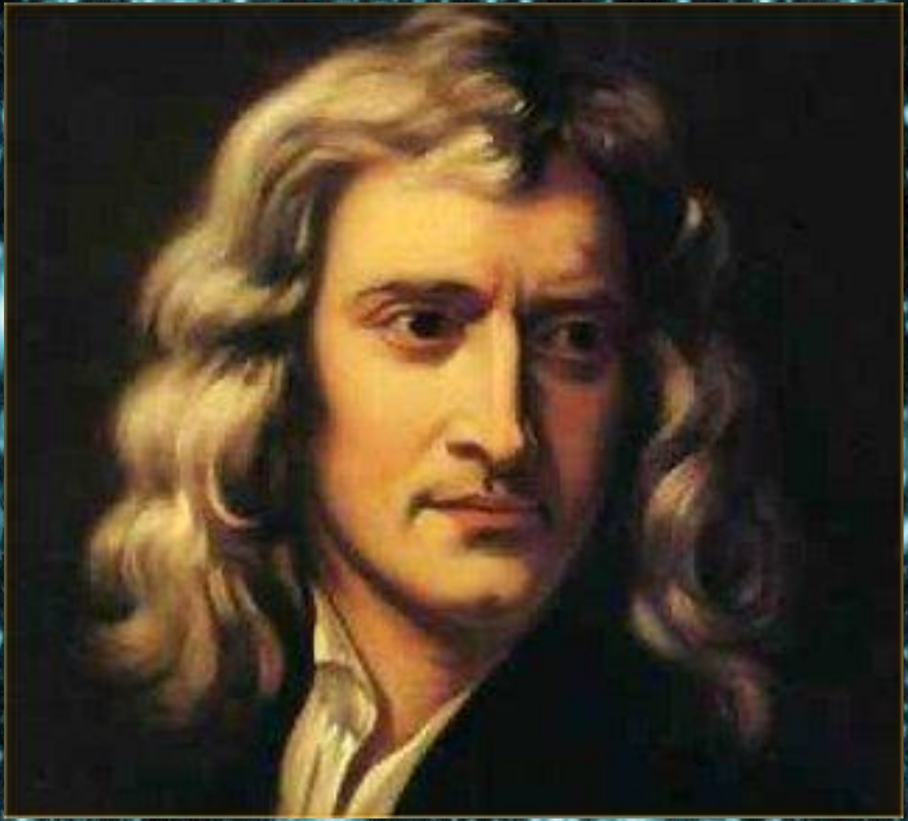
Deterministic cause-effect relation in inertial frame,

and its adaptation in a non-inertial frame!

The law of inertia enables us recognize an inertial frame of reference as one in which motion is *self-sustaining, determined entirely by initial conditions alone*.

Just where is the inertial frame?

Newton envisaged the inertial frame to be located in *deep space*, *amidst distant stars*.

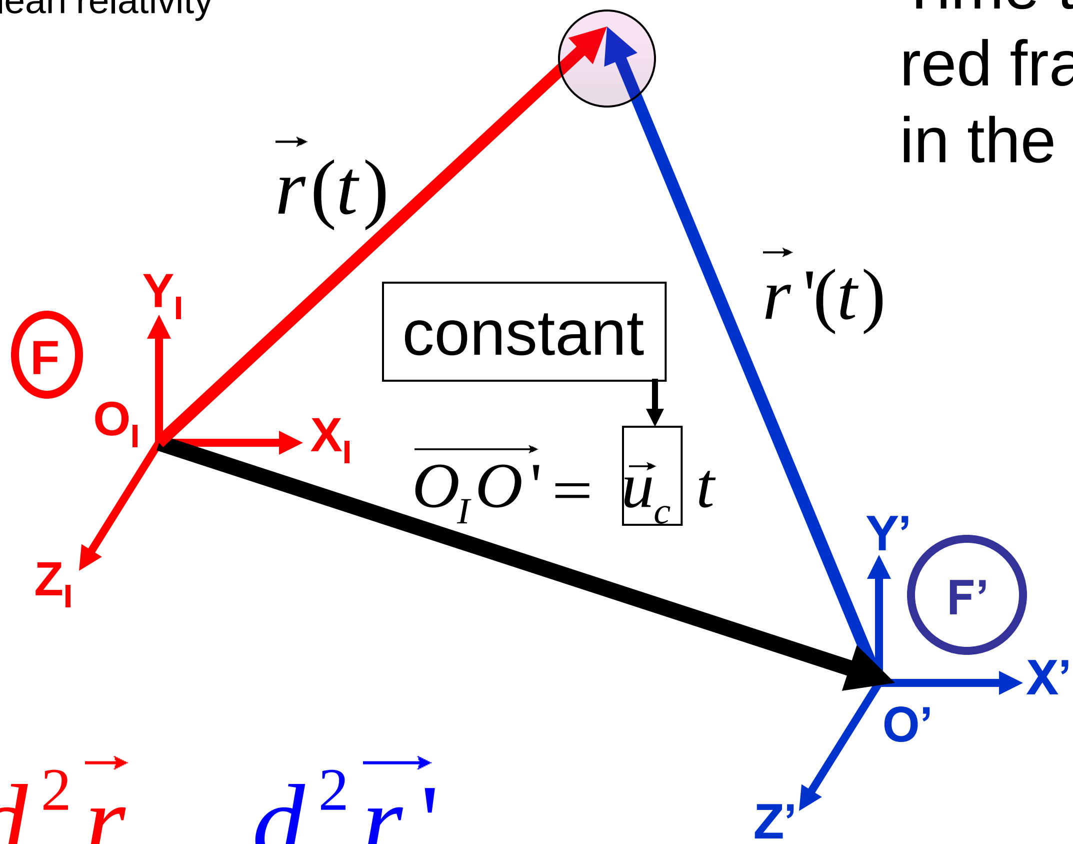


Kabhi kabhi mere dil mein,
Khayaal aata hein.....

Tu ab se pahile,
sitaron mein bas
rahi thi kahin,

Tujhe jamin pe,
bulaya gaya hei
mere liye....

Galilean relativity



Time t is the same in the red frame and in the blue frame.

$$\vec{r}(t) = \vec{r}\,'(t) + \vec{u}_c t$$

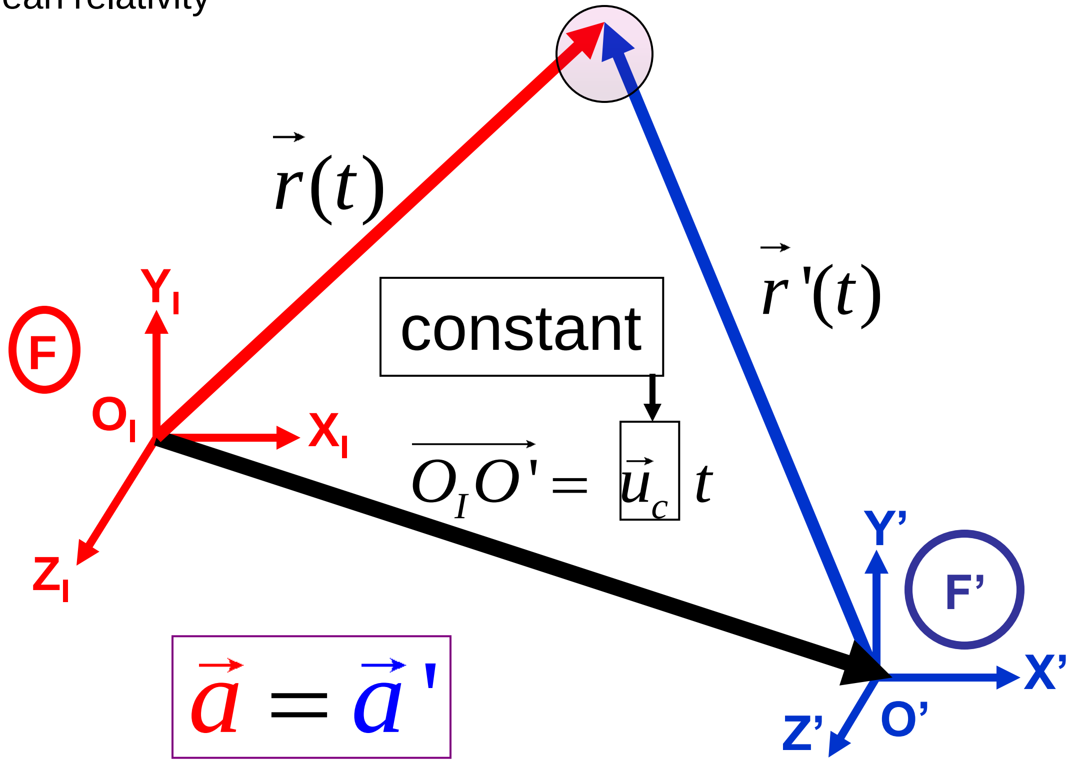
$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}\,'}{dt} + \vec{u}_c$$

First derivative with respect to time is different!

$$\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{r}\,'}{dt^2}$$

So what? Remember Galileo?

Second derivative with respect to time is, however, very much the same!



$$\frac{d^2 \vec{r}}{dt^2} = \frac{d^2 \overrightarrow{r'}}{dt^2}$$

'Acceleration', which measures departure from 'equilibrium' is essentially the same in the two frames of reference.

$$\boxed{\vec{a} = \vec{a}\,'}$$

$$\vec{F} = m\vec{a} \;\triangleleft\triangleright\; m\vec{a}\,' = \vec{F}\,'$$

Essentially the same 'cause' explains the 'effect' (acceleration) according to the same 'principle of causality'.

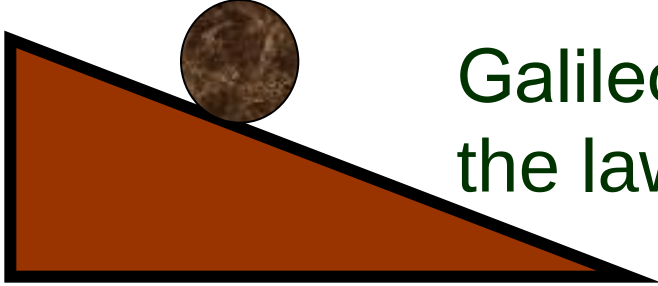
Laws of Mechanics : same in all INERTIAL FRAMES.

Galilean relativity

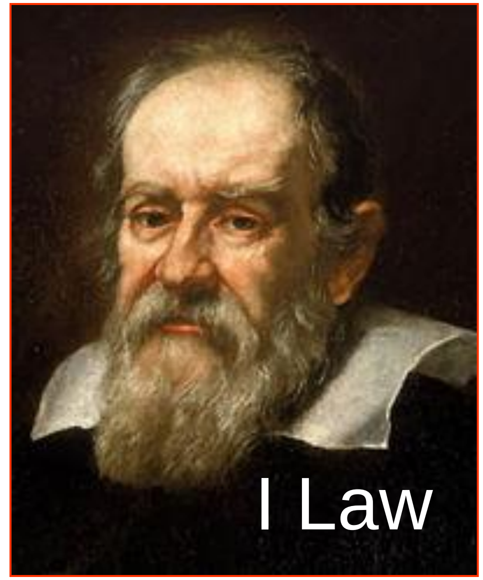
$$\overrightarrow{F} = m\vec{a} = m\vec{a}' = \overrightarrow{F}'$$

- A frame of reference moving with respect to an inertial frame of reference at constant velocity is also an inertial frame.

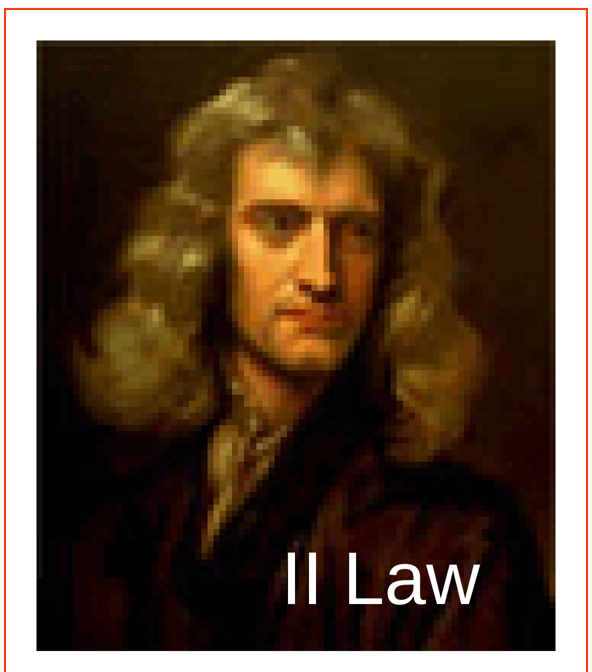
- The same force $\overrightarrow{F} = m\vec{a}$ explains the linear response (***effect/acceleration is linearly proportional to the cause/interaction***) relationship in all inertial frames.



Galileo's experiments that led him to the law of inertia.



Galileo Galilee
1564 - 1642



$\vec{F} = m\vec{a}$ Linear Response.

Effect is proportional to the *Cause*

Principle of causality.

$$\vec{F} = m\vec{a}$$ Linear Response

$$\vec{W} = m\vec{g}$$

Weight = Mass $\times$ acceleration due to gravity

Lunatic exercise!

Is lifting a cow easier on the Moon ?

THE POWER COW CRADLE

Another lunatic exercise!

Is it easier to stop a charging bull on the Moon ?

Ian Usmovets Stopping an Angry Bull (1849)

Evgraf Semenovich Sorokin (born as Kostroma Province)
1821-1892.
Russian artist and teacher, a master of historical, religious and genre paintings.

What do we mean by a CAUSE ?

CAUSE is that physical agency

which generates an EFFECT !

EFFECT: DEPARTURE FROM EQUILIBRIUM

CAUSE: 'FORCE' 'INTERACTION'

FUNDAMENTAL INTERACTION

ELECTROMAGNETIC, GRAVITATIONAL
NUCLEAR WEAK/STRONG

Galilean relativity

$$\vec{F} = m\vec{a} = m\vec{a}' = \vec{F}'$$

- A frame of reference moving with respect to an inertial frame of reference at constant velocity is also an inertial frame.
- The same force $\vec{F} = m\vec{a}$ explains the linear response (***effect/acceleration is linearly proportional to the cause/interaction***) relationship in all inertial frames.
- "An inertial frame is one in which Newton's laws hold"

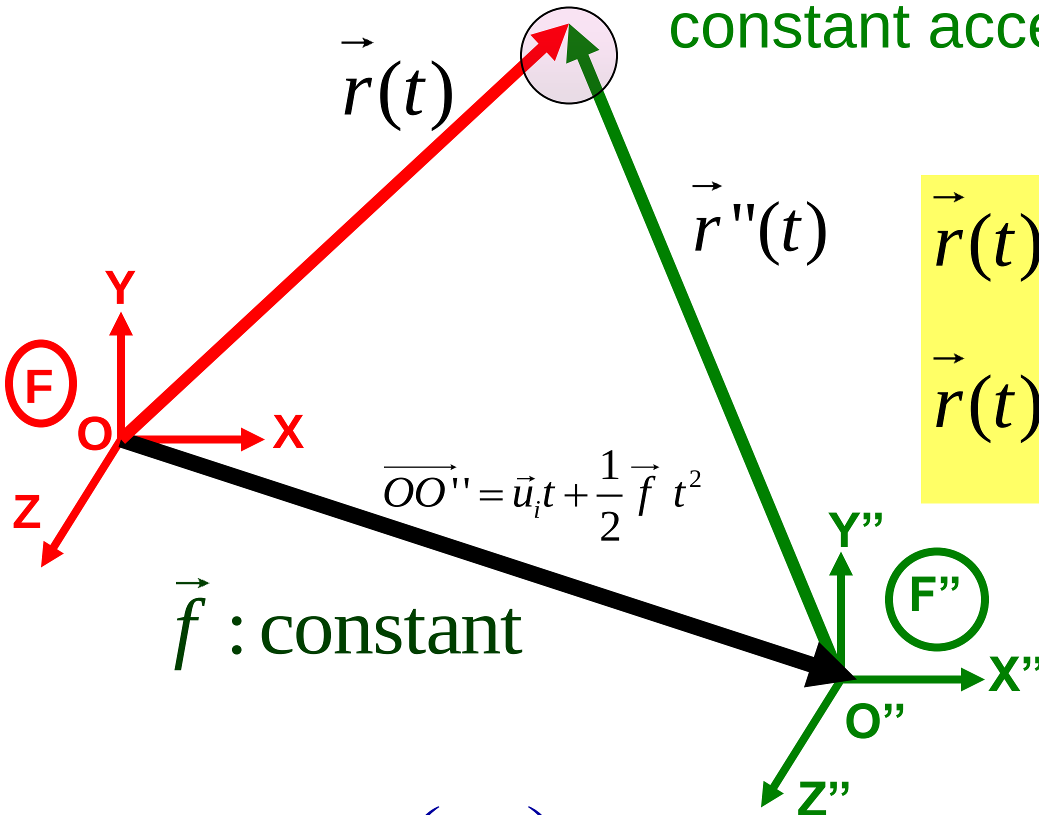
Time t is the same in the
red frame
and
in the green, double-primed frame.

Physics in an accelerated frame of reference.

What happens to the (Cause, Effect) relationship?

What happens to the Principle of Causality / Determinism?

constant acceleration $\vec{f}$.



$$\vec{r}(t) = \overrightarrow{OO''} + \vec{r}\,''(t)$$

$$\vec{r}(t) = \vec{u}_i t + \frac{1}{2}\vec{f}\ t^2 + \vec{r}\,''(t)$$

Galilean relativity

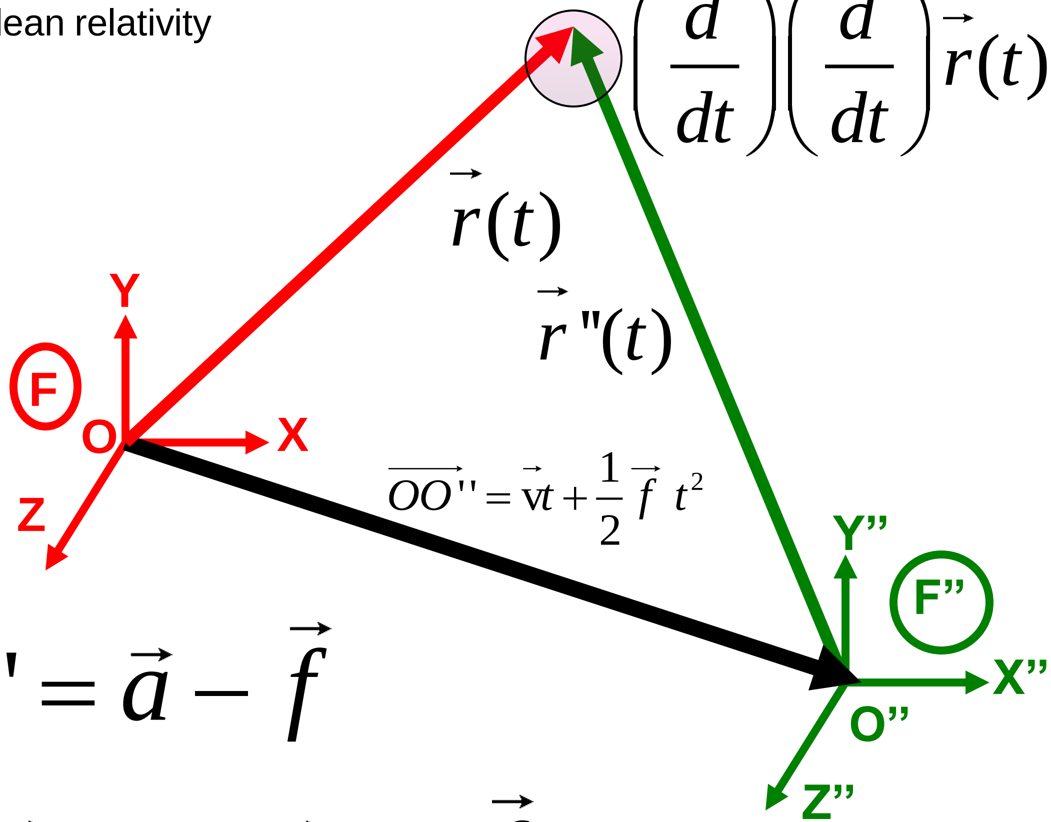
$$\left(\frac{d}{dt}\right)\vec{r}(t) = \vec{u}_i + \frac{1}{2}\vec{f}\,(2t) + \left(\frac{d}{dt}\right)\vec{r}\,''(t)$$

$$\left(\frac{d}{dt}\right)\left(\frac{d}{dt}\right)\vec{r}(t) = \vec{f} + \left(\frac{d}{dt}\right)\left(\frac{d}{dt}\right)\vec{r}\,''(t)$$

$$\vec{a} \neq \vec{a}\,''$$

Galilean relativity

$$\left(\frac{d}{dt}\right)\left(\frac{d}{dt}\right)\vec{r}(t)=\left(\frac{d}{dt}\right)\left(\frac{d}{dt}\right)\vec{r}\,''(t)+\vec{f}$$



$$\vec{a}\neq\vec{a}\,''$$

$$\vec{a}=\vec{a}\,''+\vec{f}$$

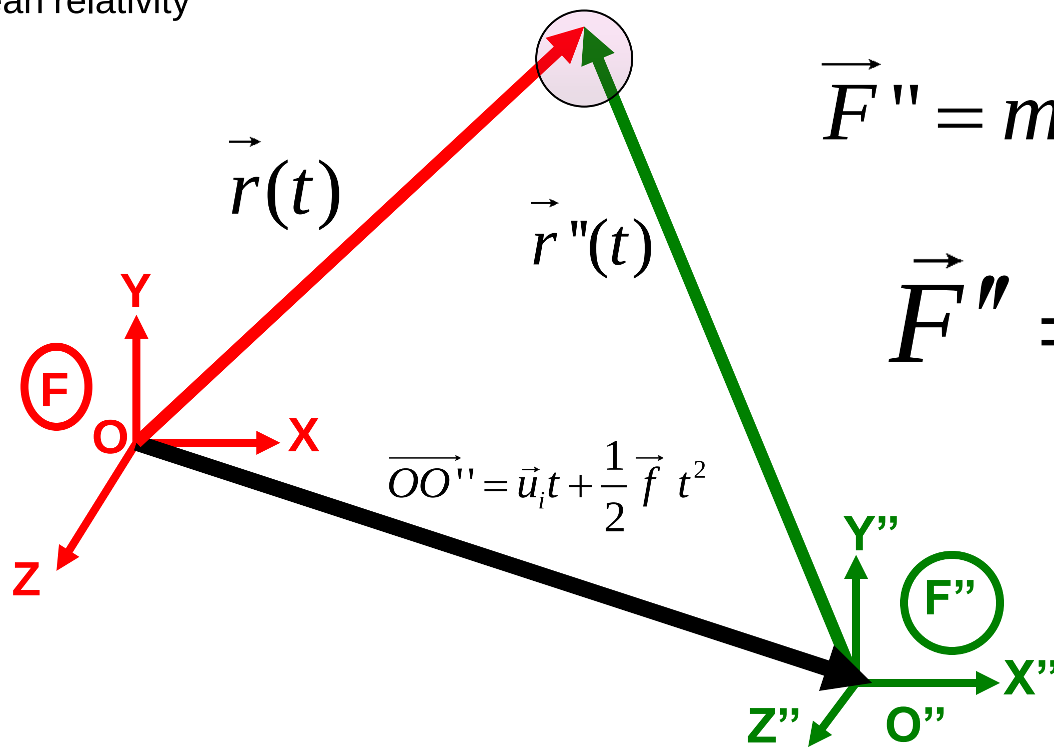
$$\vec{a}\,''=\vec{a}-\vec{f}$$

$$m\vec{a}\,''=m\vec{a}-m\vec{f}$$

$$\vec{F}\,''=m\vec{a}-m\vec{f}$$

$$\vec{F}''=\boxed{\vec{F}}-\vec{F}_{pseudo}$$

‘Real’/’Physical’ force/interaction

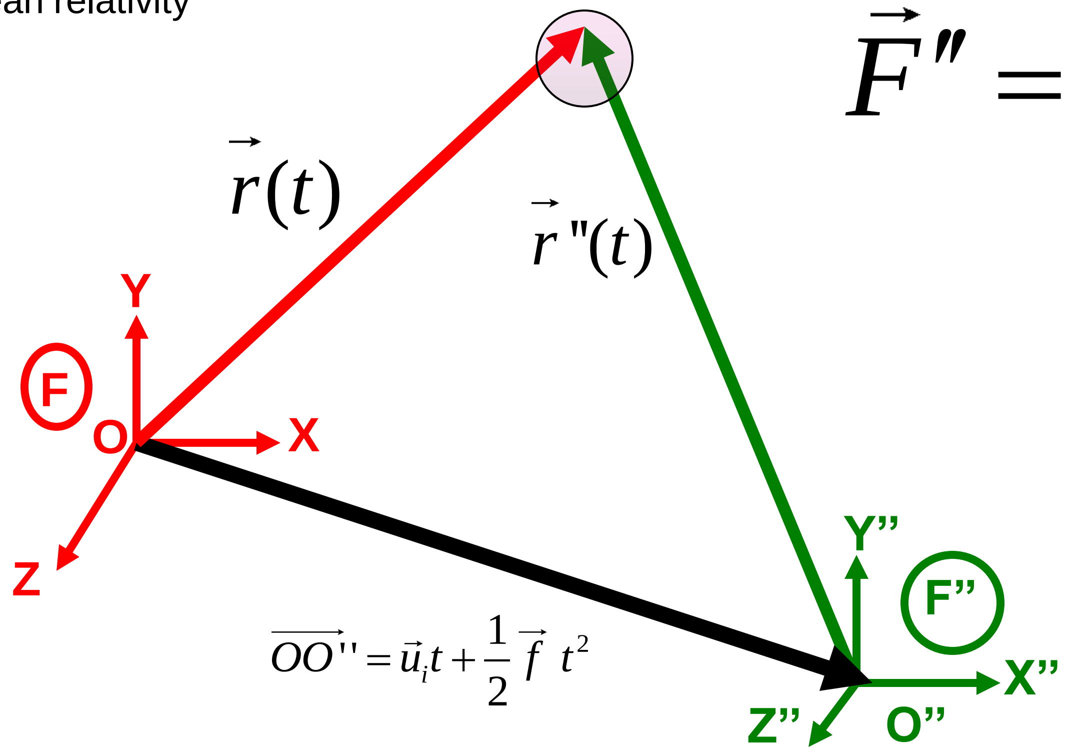


$$\overrightarrow{F''} = m\overrightarrow{a''} = m\vec{a} - m\vec{f}$$

$$\vec{F}'' = \vec{F} - \vec{F}_{pseudo}$$

Same cause-effect relationship

does **not** explain the dynamics.



$$\vec{F}'' = \overrightarrow{F} - \overrightarrow{F}_{pseudo}$$

The force/interaction which explained the acceleration in the inertial frame does not account for the acceleration in the accelerated frame of reference.

Laws of Mechanics are *not* the same in a FRAME OF REFERENCE that is accelerated with respect to an inertial frame.

$$\vec{F}'' = \overrightarrow{F} - \overrightarrow{F}_{pseudo}$$

Galilean relativity

Time t is the same in the
red frame and in the green, double-primed frame.

$$m\overrightarrow{a}'' =$$

$$\overrightarrow{F}'' = \overrightarrow{F}_{real/physical} - \overrightarrow{F}_{pseudo}$$

Real effects of pseudo-forces!

P. Chaitanya Das, G. Srinivasa Murthy, Gopal Pandurangan
and P.C. Deshmukh
Resonance, Vol. 9, Number 6, 74-85 (2004)
http://www.ias.ac.in/resonance/June2004/pdf/June2004Classroom1.pdf

Deterministic / cause-effect / relation holds only in an inertial frame of reference.

This relation inspires in our minds an intuitive perception of a fundamental interaction / force (EM, Gravity, Nuclear strong/weak).

Adaptation of causality in a non-inertial frame requires 'inventing' interactions that do not exist.

– for these 'fictitious forces', Newton's laws are of course not designed to work!

Galilean relativity:

Time t is the same in all frames of references.

RELATED ISSUES:

Weightlessness

☺ What is Einstein's weight in an elevator accelerated upward/downward?

Sergei Bubka (Ukrainian: Сергій Бубка) (born December 4, 1963) is a retired Ukrainian pole vaulter. He represented the Soviet Union before its dissolution in 1991. He is widely regarded as the best pole vaulter ever.

Reference: http://www.bookrags.com/Sergei_Bubka

http://www.stabhoch.com/bildserien/20030825_Isinbajeva_465/bildreihe.html

What enables the pole vaulting champion, Yelena Isinbayeva, twist her body in flight and clear great heights?

2008 Olympics champion: 5.05meters

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We will take a Break…

……Any questions ?

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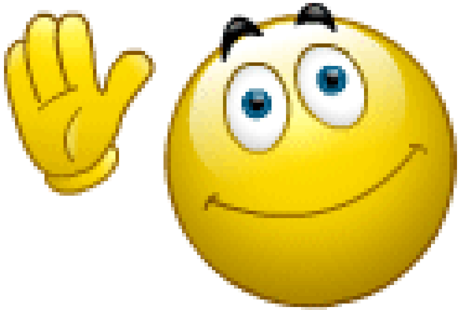

Next L16 : Non-Inertial Frame of Reference

'cause', where there isn't one!

Real effects of pseudo-forces!

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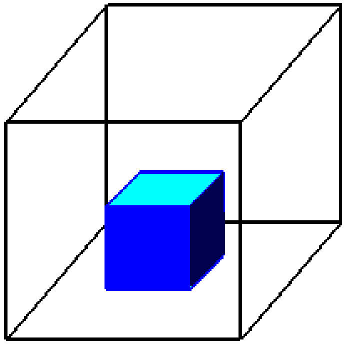
STiCM Lecture 16: Unit 5

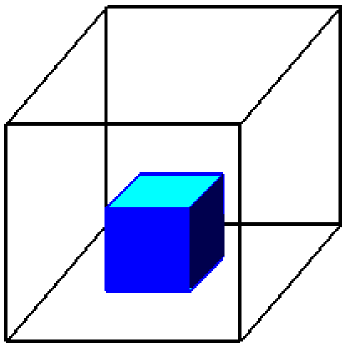
Non-Inertial Frame of Reference
‘cause’, where there isn’t one!
Real effects of pseudo-forces!

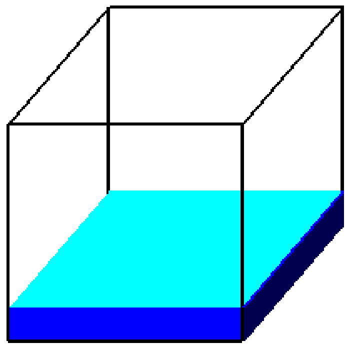


Solid

Holds Shape

Fixed Volume





Solid

Holds Shape

Fixed Volume

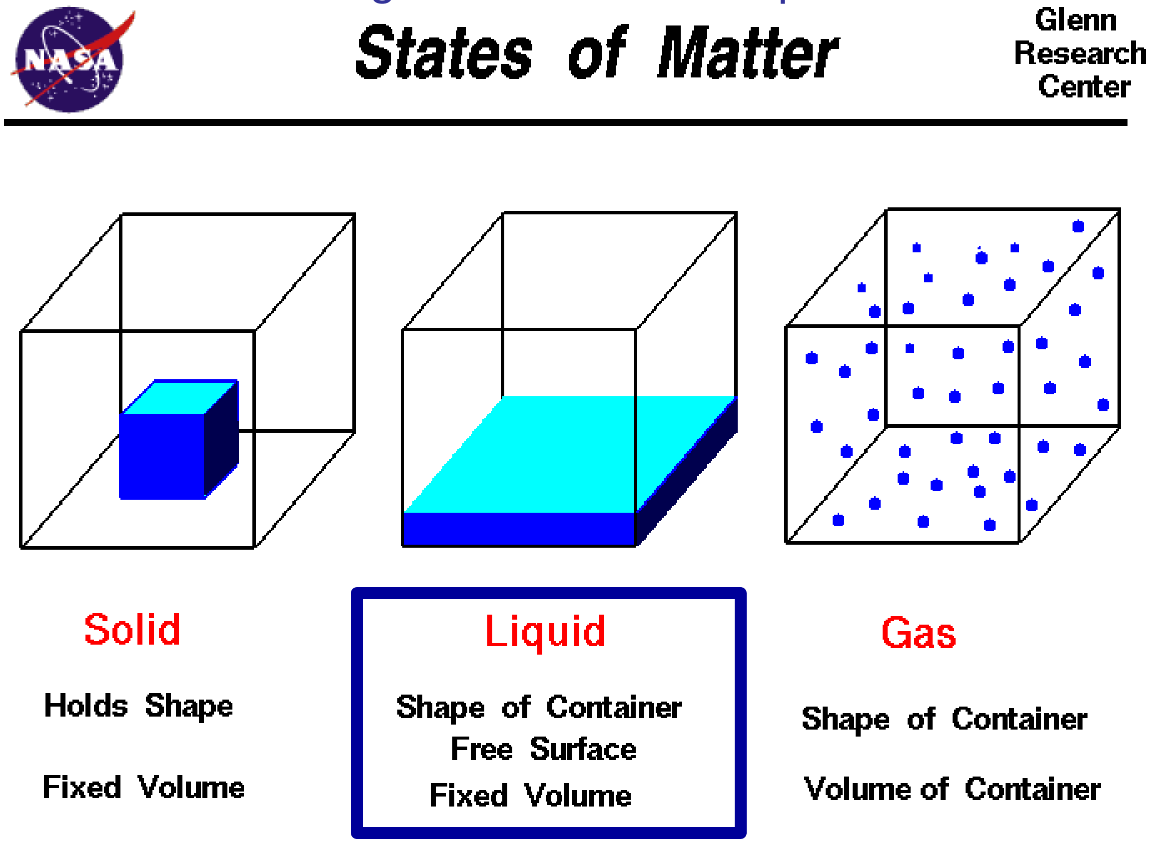
Liquid

Shape of Container

Free Surface

Fixed Volume

http://www.lerc.nasa.gov/WWW/K-12/airplane/state.html



☺ What will be the shape of a tiny little amount of a liquid in a closed (sealed) beaker when this ‘**liquid-in-a-beaker**’ system is

(a) on earth

(b) orbiting in a satellite around the earth.

In a **liquid** the molecular forces are weaker than in a solid.

A liquid takes the shape of its container with a free surface in a gravitational field.

Regardless of gravity, a liquid has a fixed volume.

Other than GRAVITY, are there other interactions that could influence the shape of the liquid's free surface?

Intra/Inter molecular forces provide the concave or the convex meniscus to the liquid. These interactions become dramatically consequential in microgravity.

The actual shape an amount of liquid will take in a closed container in microgravity depends on whether the adhesive, or the cohesive, forces are strong.

Accordingly, the liquid may form a floating ball inside, or stick to the inner walls of the container and leave a cavity inside!

You can have zero-gravity experience by booking your zero-gravity flight!

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http://www.gozerog.com/

Flight ticket (one adult) : little over $5k http://www.grc.nasa.gov/WWW/RT/RT1996/6000/6726f.htm

http://science.nasa.gov/headlines/y2000/ast12may_1.htm 29/09/09

On Earth, gravity-driven buoyant convection causes a candle flame to be teardrop-shaped and carries soot to the flame's tip, making it yellow.

In microgravity, where convective flows are absent, the flame is spherical, soot-free, and blue.

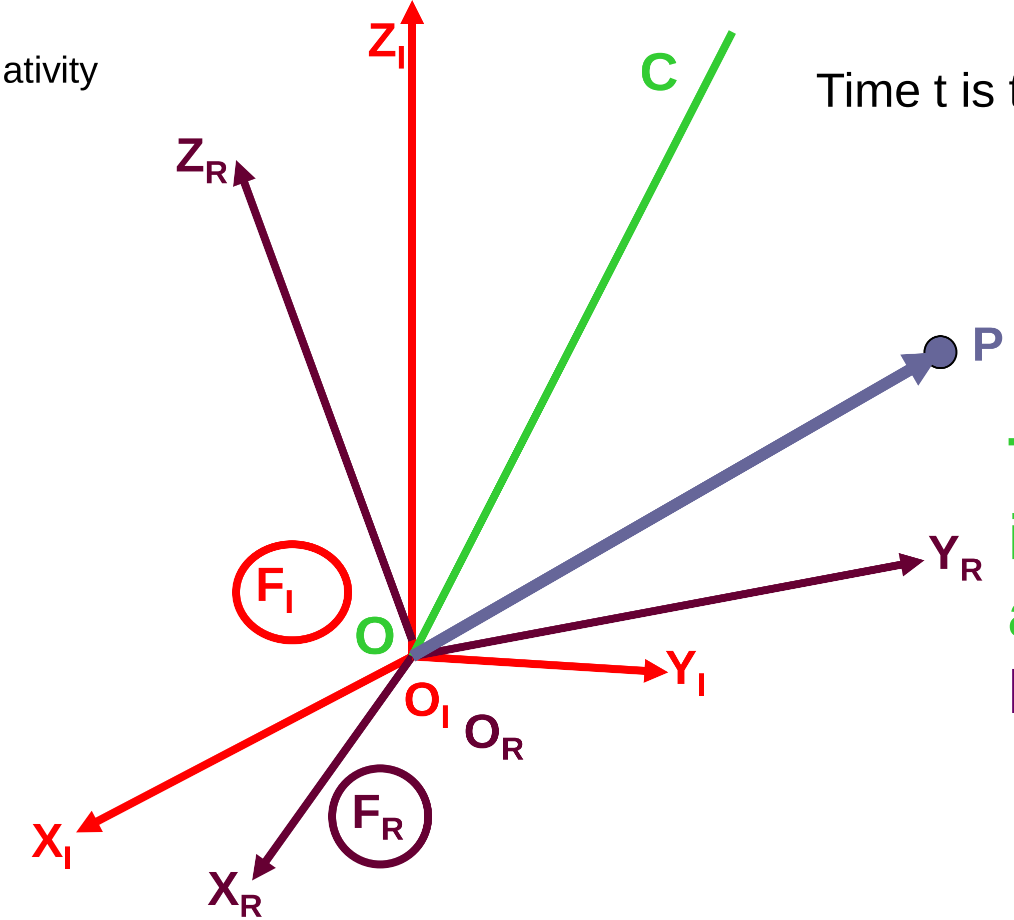
"Micro gravity" ?

$$G\frac{m_1 m_2}{r^2}$$

There is no 'insulation' from gravity!

Observations in a rotating frame of reference.

Galilean relativity



Time t is the same in both the frames

The green axis OC is some arbitrary axis about which F_R is rotating.

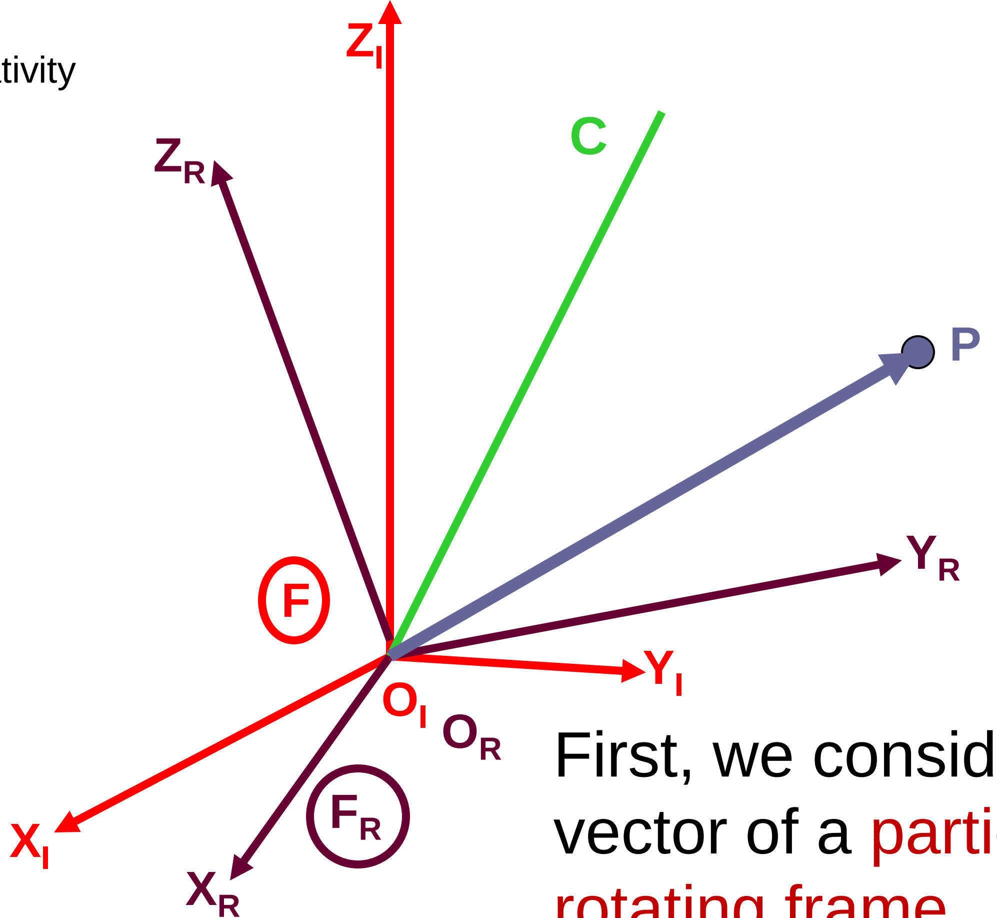
Clearly, the coordinates/<u>components</u> of the point P in the inertial frame of reference are different from those in the rotating frame of reference.

We shall ignore translational motion of the new frame relative to the inertial frame F_I.

We have already considered that.

We can always superpose the two (translational and rotating) motions.

Galilean relativity

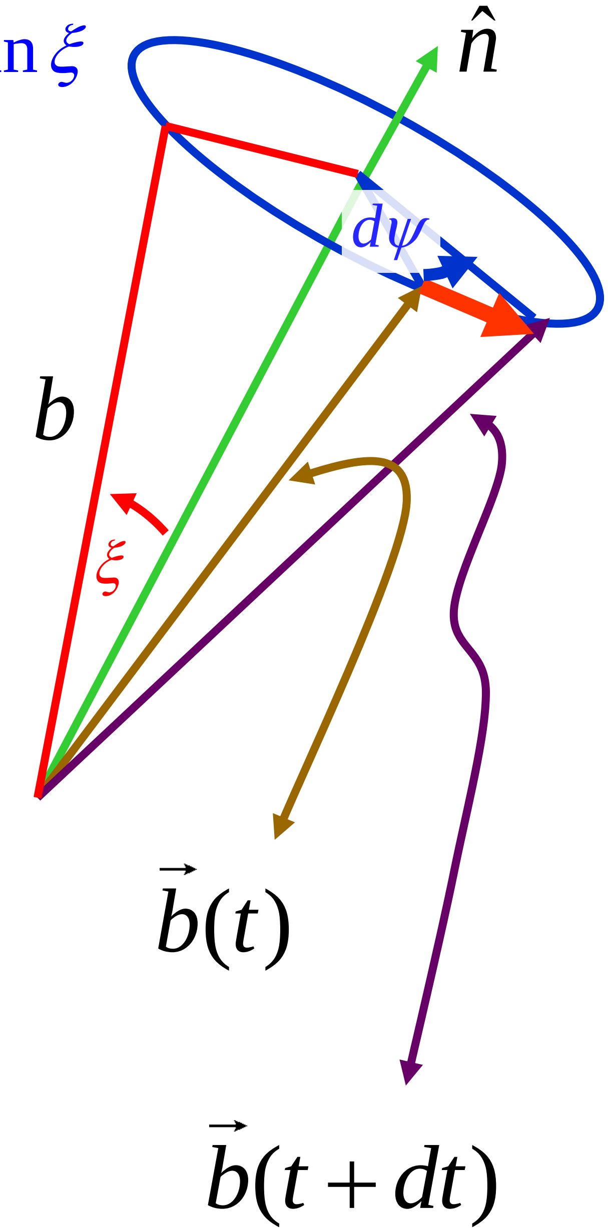


First, we consider the position vector of a particle at rest in the rotating frame.

Clearly, its time-derivative in the rotating frame is zero, but not so in the inertial frame.

$$\boxed{\left(\frac{d}{dt}\right)_I \neq \left(\frac{d}{dt}\right)_R}$$

$$\left(\frac{\mathrm{d}}{\mathrm{dt}}\right)_R \vec{b} \neq \left(\frac{\mathrm{d}}{\mathrm{dt}}\right)_I \vec{b}$$



The particle seen at 'rest' in the rotating frame would appear to have moved to a new location as seen by an observer in F_I.

Galilean relativity

Time t is the same in the red frame and in the purple, rotating, frame.

NOTE!

The time-derivative is different in the rotating frame.

$$\left(\frac{d}{dt}\right)_I \neq \left(\frac{d}{dt}\right)_R$$

Often, one uses the term
'SPACE-FIXED FRAME OF REFERENCE' for F_I, and
'BODY-FIXED FRAME OF REFERENCE' for F_R.

We shall develop our analysis for an arbitrary vector $\vec{b}$, the only condition being that it is itself not a time-derivative in the rotating frame of some another vector.

$$\textit{No}\text{ vector } \vec{\mathrm{q}} \text{ exists such that } \left(\frac{\mathrm{d}}{\mathrm{dt}}\right)_R \vec{q} = \vec{b}$$

$$\left(\frac{\mathrm{d}}{\mathrm{dt}}\right)_R \vec{b} = \vec{0} \text{ in the rotating frame } \mathrm{F_R}.$$

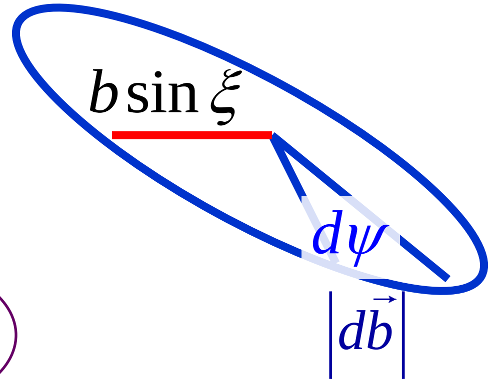
$$\textit{Question}: \text{ What is } \left(\frac{\mathrm{d}}{\mathrm{dt}}\right)_I \vec{b}\ ?$$

$$d\vec{b} = \vec{b}(t+dt) - \vec{b}(t) = |d\vec{b}|\,\hat{u}$$

where $\hat{u} = \dfrac{\hat{n}\times\hat{b}}{\left|\hat{n}\times\hat{b}\right|}.$

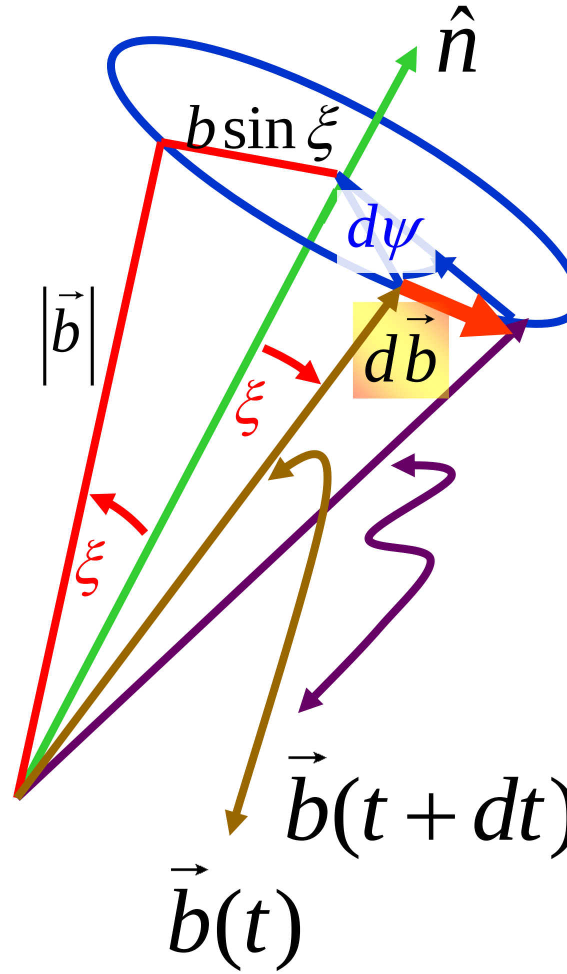
$$\xi = \angle(\hat{n},\hat{b})$$

$$\left|d\vec{b}\right| = (b\sin\xi)(d\psi)$$



$$d\vec{b} = (b\sin\xi)(d\psi)\frac{\hat{n}\times\hat{b}}{\left|\hat{n}\times\hat{b}\right|}$$

These two terms are equal and hence cancel.



$$d\vec{b} = d\psi\,\hat{n}\times\vec{b}$$

$$d\vec{b} = (\vec{\omega}dt)\times\vec{b}$$

since $\vec{\omega} = \dfrac{d\psi}{dt}\hat{n}$

$$\Rightarrow \left(\frac{d}{dt}\right)_I \vec{b} = \vec{\omega}\times\vec{b}$$

$$\left(\frac{d}{dt}\right)_I \vec{b} = \boxed{\vec{\omega} \times \vec{b}}$$

Remember! The vector $\vec{b}$ itself did not have any time-dependence in the rotating frame.

If $\vec{b}$ has a time dependence in the rotating frame, the following operator equivalence would follow:

$$\left(\frac{d}{dt}\right)_I \vec{b} = \boxed{\vec{\omega} \times \vec{b}} + \boxed{\left(\frac{d}{dt}\right)_R \vec{b}}$$

Operator Equivalence: $$\left(\frac{d}{dt}\right)_I = \boxed{\left(\frac{d}{dt}\right)_R} + \boxed{\vec{\omega} \times}$$

$$\left(\frac{d}{dt}\right)_I = \left(\frac{d}{dt}\right)_R + \vec{\omega}\times$$

$$\left(\frac{d}{dt}\right)_I \vec{r} = \left(\frac{d}{dt}\right)_R \vec{r} + \vec{\omega}\times\vec{r}$$

Operating twice:

$$\left(\frac{d}{dt}\right)_I \left(\frac{d}{dt}\right)_I \vec{r} = \left(\frac{d}{dt}\right)_R \left\{\left(\frac{d}{dt}\right)_R \vec{r} + \vec{\omega}\times\vec{r}\right\} + \\ + \vec{\omega}\times\left\{\left(\frac{d}{dt}\right)_R \vec{r} + \vec{\omega}\times\vec{r}\right\}$$

$$\left(\frac{d^2}{dt^2}\right)_I \vec{r} = \left(\frac{d^2}{dt^2}\right)_R \vec{r} + \left(\frac{d}{dt}\right)_R \left(\vec{\omega}\times\vec{r}\right) + \vec{\omega}\times\left(\frac{d}{dt}\right)_R \vec{r} \\ + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$$

Multiplying by mass ‘m’, we shall get quantities that have dimensions of ‘force’.

$$m\left(\frac{d^2}{dt^2}\right)_R \vec{r} = m\left(\frac{d^2}{dt^2}\right)_I \vec{r} - m\left(\frac{d\vec{\omega}}{dt}\right)_R \times \vec{r} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r} +$$

$$- \mathrm{m}\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$$

$$\vec{F}_R = \vec{F}_I \boxed{-\vec{F}_{\dot{\omega}}} \boxed{-2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}} \boxed{-\mathrm{m}\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)}$$

‘Leap second’ term

‘Coriolis force’

‘Centrifugal force’

Gaspard Gustave
de Coriolis term
1792 - 1843

We will take a Break…

……Any questions ?

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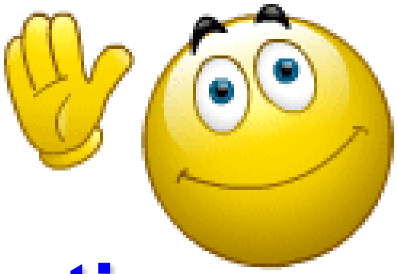
Next L17 : Coriolis Deflection

Foucault Pendulum

Cyclonic storm's direction

Real Effects of Pseudo-forces!



$$\vec{F}_R = \vec{F}_I - \vec{F}_{\dot{\omega}} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r} - \mathrm{m}\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

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'cause', when there really isn't one!
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‘force’ / ‘fundamental interaction’ in the inertial frame

$$m\left(\frac{d^2}{dt^2}\right)_R \vec{r} = m\left(\frac{d^2}{dt^2}\right)_I \vec{r} - m\left(\frac{d\vec{\omega}}{dt}\right)_R \times \vec{r} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r} - m\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$$

Pseudoforce in the rotating frame

REAL EFFECTS OF PSEUDOFORCES !

$$\vec{F}_R = \vec{F}_I - \vec{F}_{\dot{\omega}} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r} - m\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$$

‘Leap second’ term

‘Coriolis force’

‘Centrifugal force’

Why are Leap Seconds Used?

The time taken by the earth to do one rotation differs from day to day and from year to year.

The Earth was slower than atomic clocks by 0.16 seconds in 2005;

by 0.30 seconds in 2006;

by 0.31 seconds in 2007;

and by 0.32 seconds in 2008.

It was only 0.02 seconds slower in 2001.

http://www.timeanddate.com/time/leapseconds.html ; 14th October, 2009

The atomic clocks can be reset to add an extra second, known as the leap second, to synchronize the atomic clocks with the Earth's observed rotation.

The most accurate and stable time comes from atomic clocks but for navigation and astronomy purposes, *but it is the atomic time that is synchronized with the Earth's rotation.*

‘Leap second’ term

The International Earth Rotation and Reference System Service (IERS) decides when to introduce a leap second in UTC (**C**oordinated **U**niversal **T**ime).

On one average day, the difference between atomic clocks and Earth's rotation is around 0.002 seconds, or around 1 second every 1.5 years.

IERS announced on July 4, 2008, that a leap second would be added at 23:59:60 (or near midnight) UTC on December 31, 2008.

This was the 24th leap second to be added since the first leap second was added in 1972.

http://www.timeanddate.com/time/leapseconds.html ; 14th October, 2009

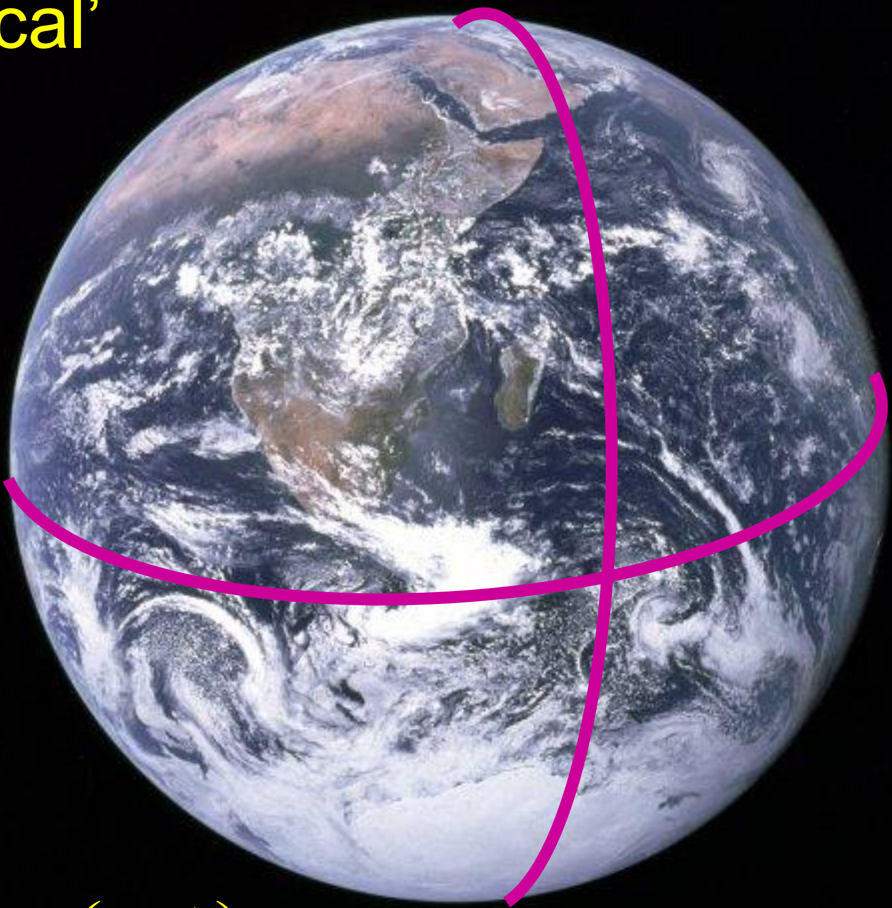
Let us consider 3 ‘definitions’ of the ‘vertical’

[1] ‘vertical’ is defined by the radial line from the center of the earth to a point on the earth’s surface

[2] ‘vertical’ is defined by a ‘plumb line’ suspended at the point under consideration.

[3] ‘vertical’ is defined by the space curve along which a chalk falls, if you let it go!

3 ‘definitions’ of the ‘vertical’



$$m\left(\frac{d^2}{dt^2}\right)_R \vec{r} = m\left(\frac{d^2}{dt^2}\right)_I \vec{r} - m\left(\frac{d\vec{\omega}}{dt}\right)_R \times \vec{r} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

$$- \mathrm{m}\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$$

7.2921159 $\times 10^{-5}$
radians per second

$$-m\left(\frac{d\vec{\omega}}{dt}\right)_R \times \vec{r}$$

$$m\left(\frac{d^2}{dt^2}\right)_R \vec{r} = m\left(\frac{d^2}{dt^2}\right)_I \vec{r} - m\left(\frac{d\vec{\omega}}{dt}\right)_R \times \vec{r} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$
$$- \mathrm{m}\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$$

http://commons.wikimedia.org/wiki/File:Globespin.gif

Earth and moon seen from MARS

First image of Earth & Moon,

ever taken from another planet,

from Mars,

by MGS,

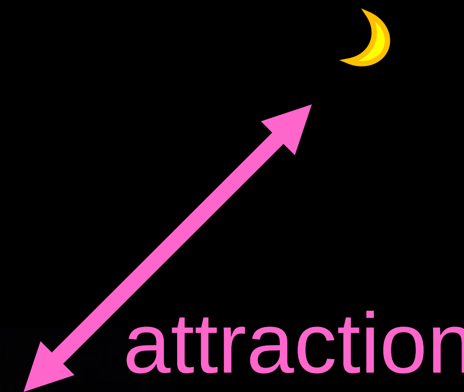
on 8 May 2003 at 13:00 GMT.

MARS GLOBAL SURVEYOR

http://mars8.jpl.nasa.gov/mgs/

http://space.about.com/od/pictures/ig/Earth-Pictures-Gallery/Earth-and-Moon-as-viewed-from-.htm

The moon is attracted toward the earth due to their mutual gravitational attraction.



Will there be an eventual collision?

If so, when?

If not, why not?

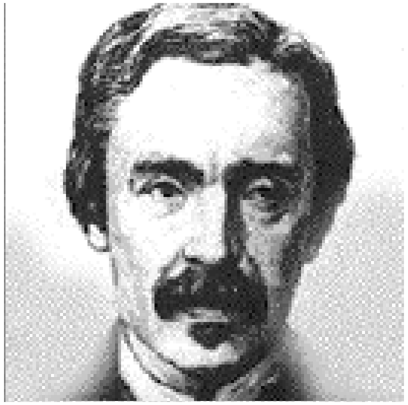
[1] When travelling in a car/bus, you experience what you call as a 'centrifugal force'.

Which physical agency exerts it ?

Is there a corresponding force of reaction?

[2] How does a centrifuge work ?

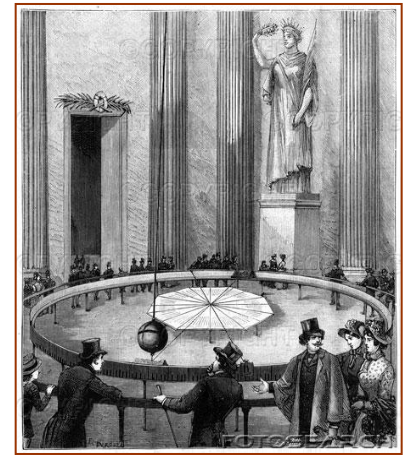
[3] Do the centripetal and the centrifugal forces constitute an 'action - reaction' pair of Newton's 3rd law?



“ Foo-Koh”

The plane of oscillation of the Foucault pendulum is seen to rotate due to the Coriolis effect.

The plane rotates through one full rotation in 24 hours at poles, and in ~33.94 hours at a latitude of 45^0 (Latitude of Paris is ~49^0).



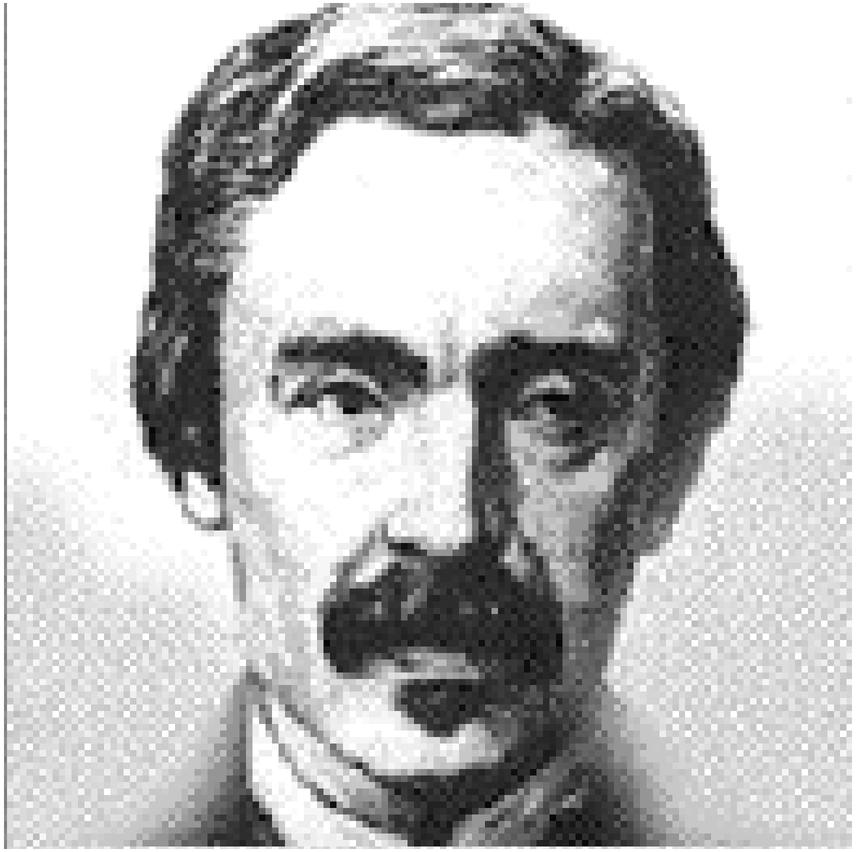
$$\vec{F}_R = \vec{F}_I \boxed{-\vec{F}_{\dot{\omega}}} \boxed{-2m\vec{\omega}\times\left(\frac{d}{dt}\right)_R \vec{r}} \boxed{-\ \mathrm{m}\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)}$$

‘Leap second’ term

‘Coriolis force’

‘Centrifugal force’

Gaspard Gustave de Coriolis
1792 - 1843



Jean Bernard Léon Foucault
1819 - 1868

$$m\left(\frac{d^2}{dt^2}\right)_R \vec{r} = m\left(\frac{d^2}{dt^2}\right)_I \vec{r}$$

$$-m\left(\frac{d\vec{\omega}}{dt}\right)_R \times \vec{r}$$

$$-2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

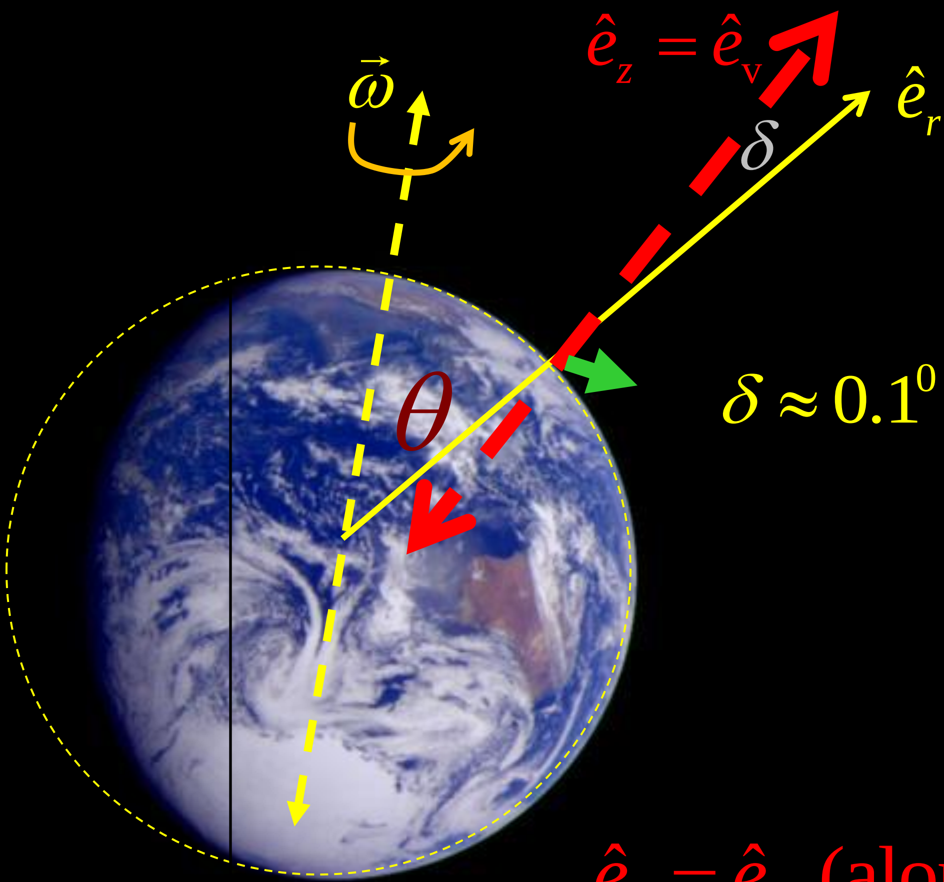
$$-\ m\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$$

We often ignore the 'leap second' and the centrifugal term.

Pseudo Forces

$$\vec{F}_{centrifugal} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r}) = -m\omega^2 r\hat{e}_\omega \times (\hat{e}_\omega \times \hat{e}_r)$$

$$= -m\omega^2 r\hat{e}_\omega \times (\sin\theta\hat{e}_\varphi) = m\omega^2 r\sin\theta\hat{e}_\rho$$



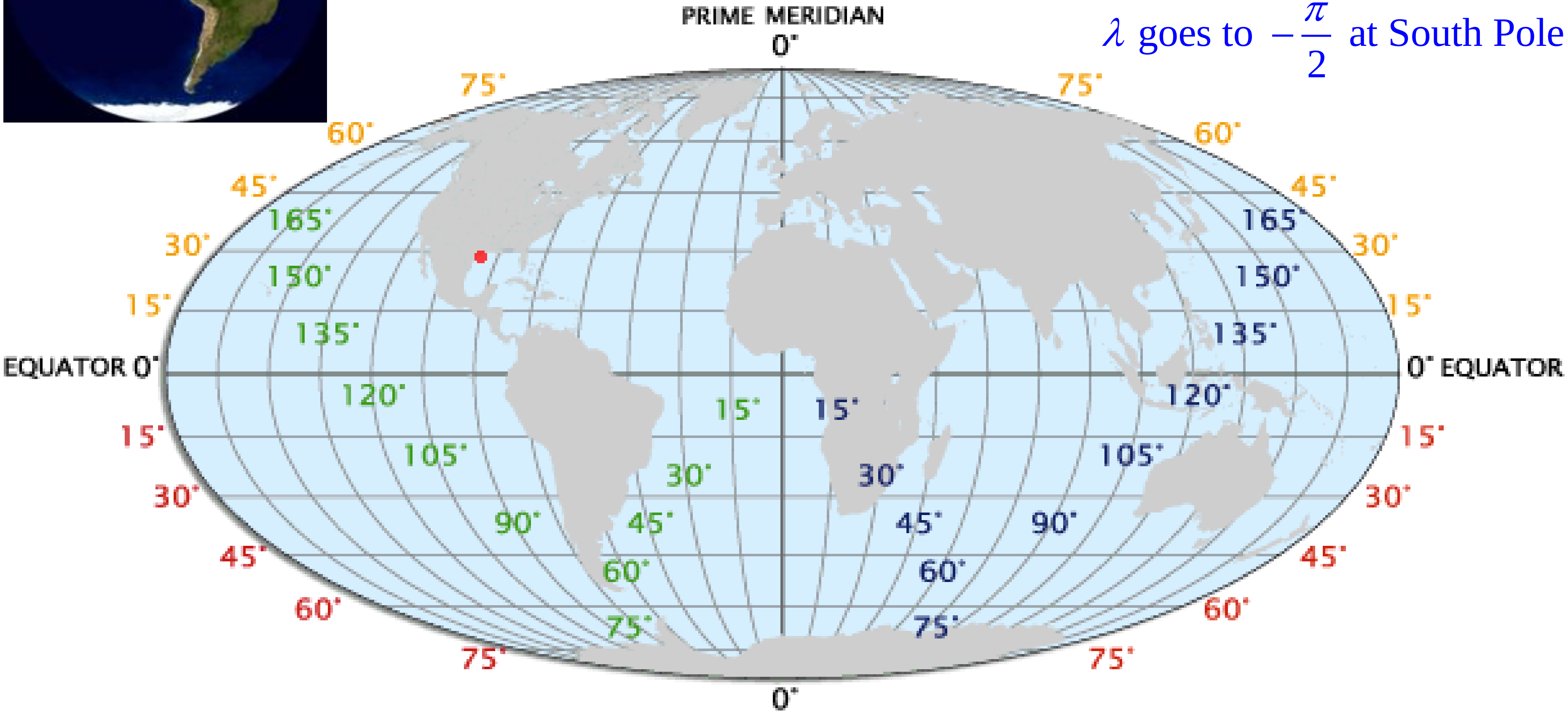
$\delta \approx 0.1^0$ at latitude of 45^0

$\hat{e}_z = \hat{e}_v$ (along 'local vertical')

$$\lambda = \frac{\pi}{2} \text{ at North Pole}$$

$$\lambda = 0 \text{ at Equator}$$

$$\lambda \text{ goes to } -\frac{\pi}{2} \text{ at South Pole}$$



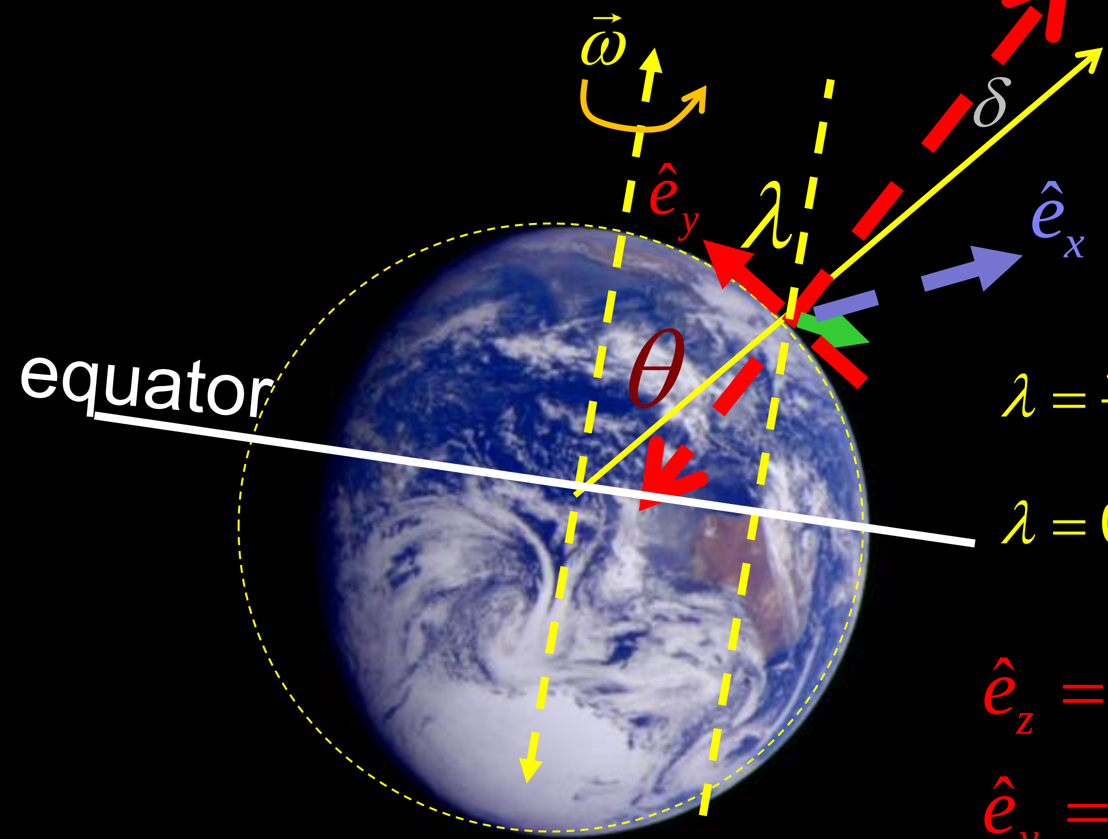
Terrestrial equatorial radius is 6378 km.

Polar radius is 6357 km.

$$\vec{F}_{Coriolis} = -2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

$$\boxed{\vec{\omega} = \left(\vec{\omega} \bullet \hat{e}_y\right)\hat{e}_y + \left(\vec{\omega} \bullet \hat{e}_z\right)\hat{e}_z}$$

$$\hat{e}_z = \hat{e}_v$$



$$\hat{e}_x = \hat{e}_{East} = \hat{e}_y \times \hat{e}_z$$

$\lambda = \frac{\pi}{2}$ at North Pole

$\lambda = 0$ at Equator

$$\lambda = \measuredangle\left(\vec{\omega}, \hat{e}_{North}\right) = \measuredangle\left(\vec{\omega}, \hat{e}_y\right)$$

$\hat{e}_z = \hat{e}_v$ ('local vertical')

$\hat{e}_y = \hat{e}_{North}$

$\hat{e}_x = \hat{e}_{East} = \hat{e}_y \times \hat{e}_z$

$$\vec{F}_{Coriolis} = -2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

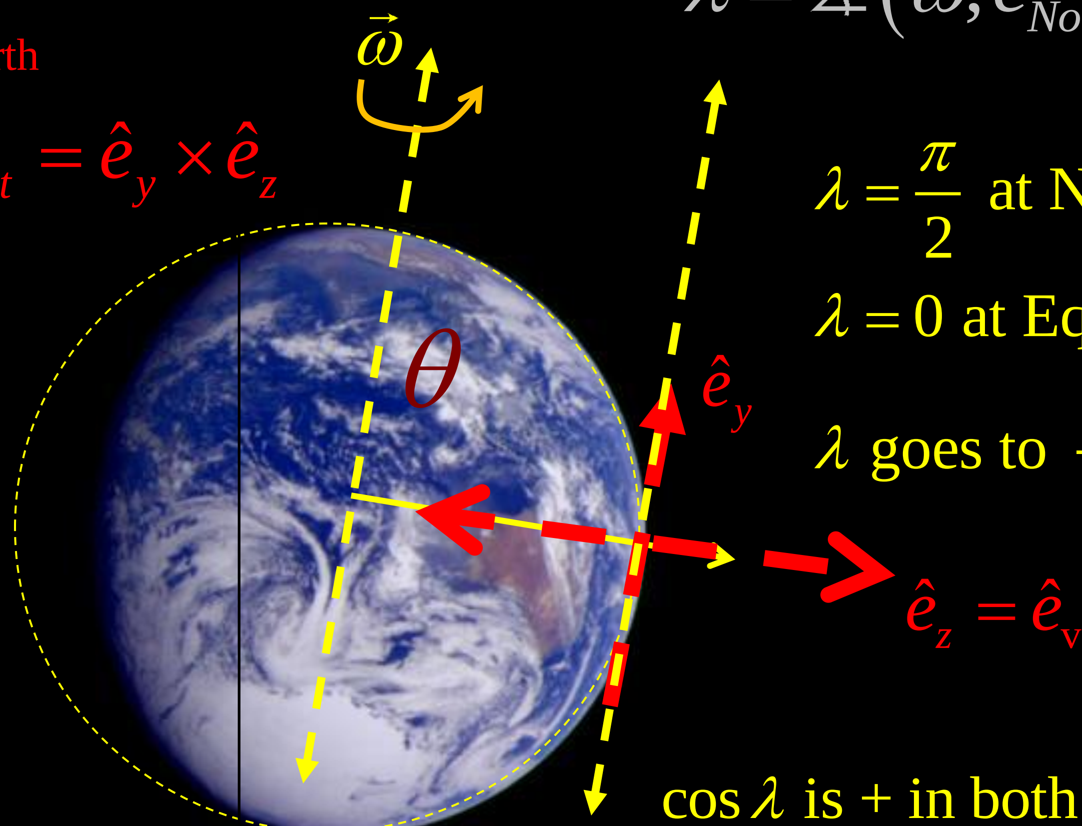
$$\boxed{\vec{\omega} = \left(\vec{\omega} \bullet \hat{e}_y\right)\hat{e}_y + \left(\vec{\omega} \bullet \hat{e}_z\right)\hat{e}_z}$$

$\hat{e}_z = \hat{e}_v$ ('local vertical')

$\hat{e}_y = \hat{e}_{\text{North}}$

$\hat{e}_x = \hat{e}_{East} = \hat{e}_y \times \hat{e}_z$

$$\lambda = \measuredangle\left(\vec{\omega}, \hat{e}_{North}\right) = \measuredangle\left(\vec{\omega}, \hat{e}_y\right)$$



$\lambda = \dfrac{\pi}{2}$ at North Pole

$\lambda = 0$ at Equator

λ goes to $-\dfrac{\pi}{2}$ at South Pole

$\cos\lambda$ is + in both N- and S-hemispheres

$$\vec{F}_{Coriolis} = -2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

$$\vec{\omega} = \left(\vec{\omega} \bullet \hat{e}_y\right)\hat{e}_y + \left(\vec{\omega} \bullet \hat{e}_z\right)\hat{e}_z$$

Coriolis deflection of an object in 'free' 'fall' at a point on earth's surface:

$$\left(\frac{d}{dt}\right)_R \vec{r} = \vec{\mathrm{v}}_R = \mathrm{v}_R(-\hat{e}_z)$$

$$\vec{F}_{Coriolis} = -2m\left[\left(\vec{\omega} \bullet \hat{e}_y\right)\hat{e}_y + \left(\vec{\omega} \bullet \hat{e}_z\right)\hat{e}_z\right] \times \mathrm{v}_R(-\hat{e}_z)$$

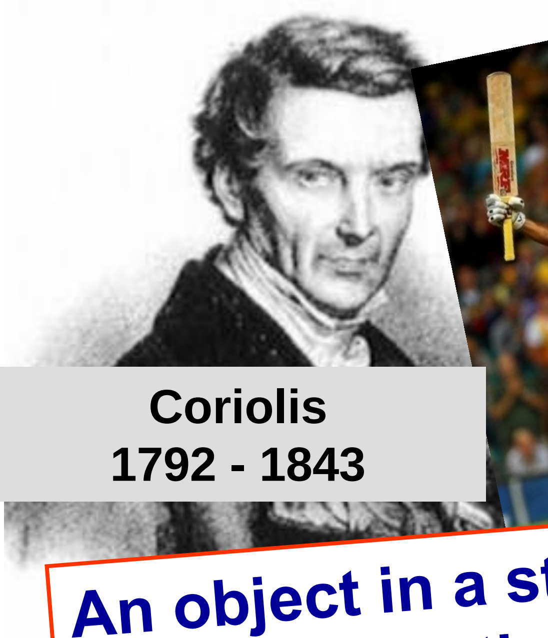
$$= 2m\left(\vec{\omega} \bullet \hat{e}_y\right)\hat{e}_x = 2m\omega\cos\lambda\hat{e}_{East}$$

$$0 \le \lambda \le \frac{\pi}{2} \ (N \ hemisphere)$$

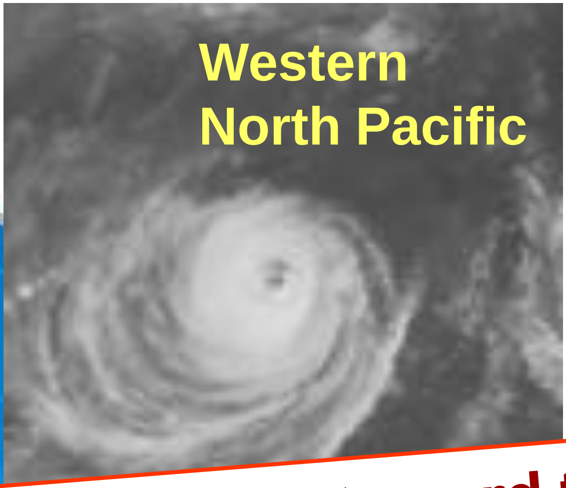
$$-\frac{\pi}{2} \le \lambda \le 0 \ (S \ hemisphere)$$

Coriolis deflection: toward East in both the Northern & the Southern Hemispheres

$\cos\lambda$ is + in both N- and S-hemispheres

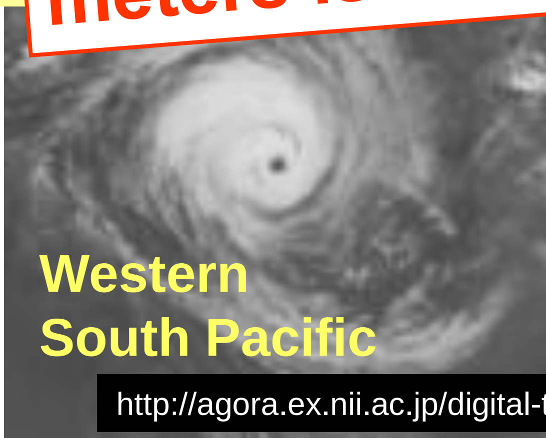


Coriolis
1792 - 1843



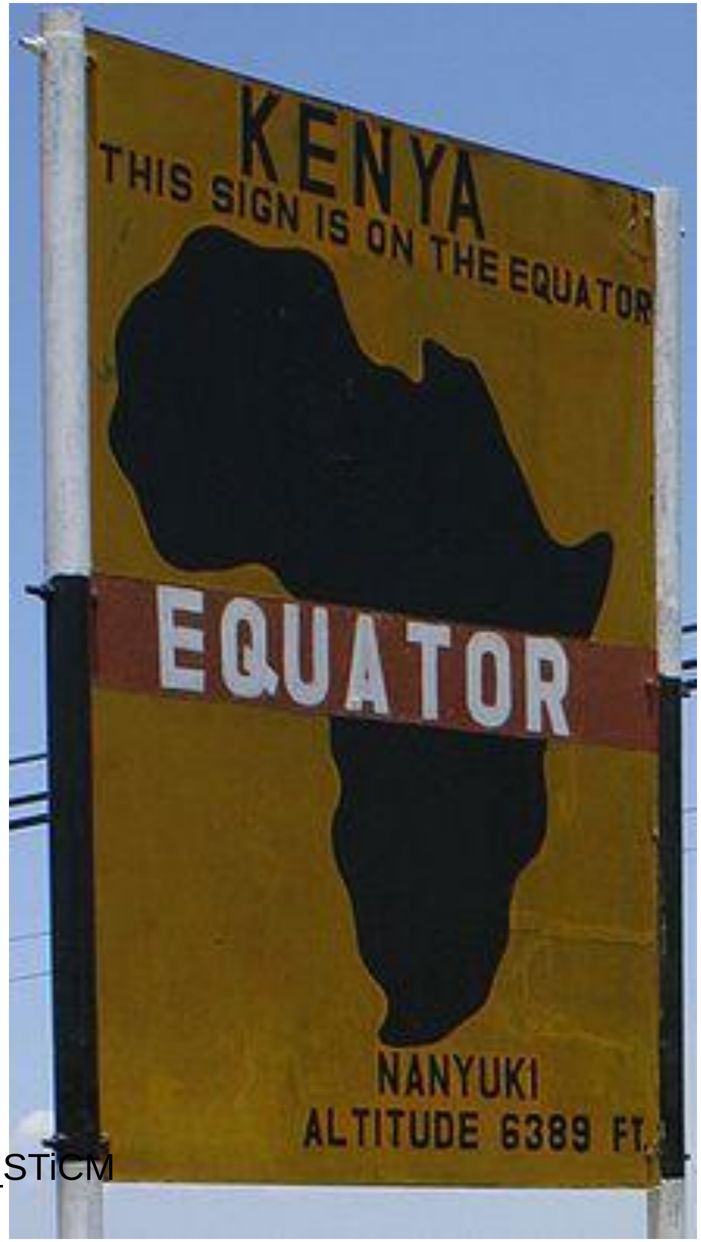
An object in a state of free fall gets deflected **toward the East**, in both the NORTHERN and the SOUTHERN hemispheres!

At a latitude of 60^0 an object falling through 100 meters is deflected through ~1 cm.



http://agora.ex.nii.ac.jp/digital-typhoon/start_som_wnp.html.en http://www.youtube.com/watch?v=ZvLYrZ3vgio&NR=1

This Nanyuki tourist attraction is *not* due to Coriolis effect



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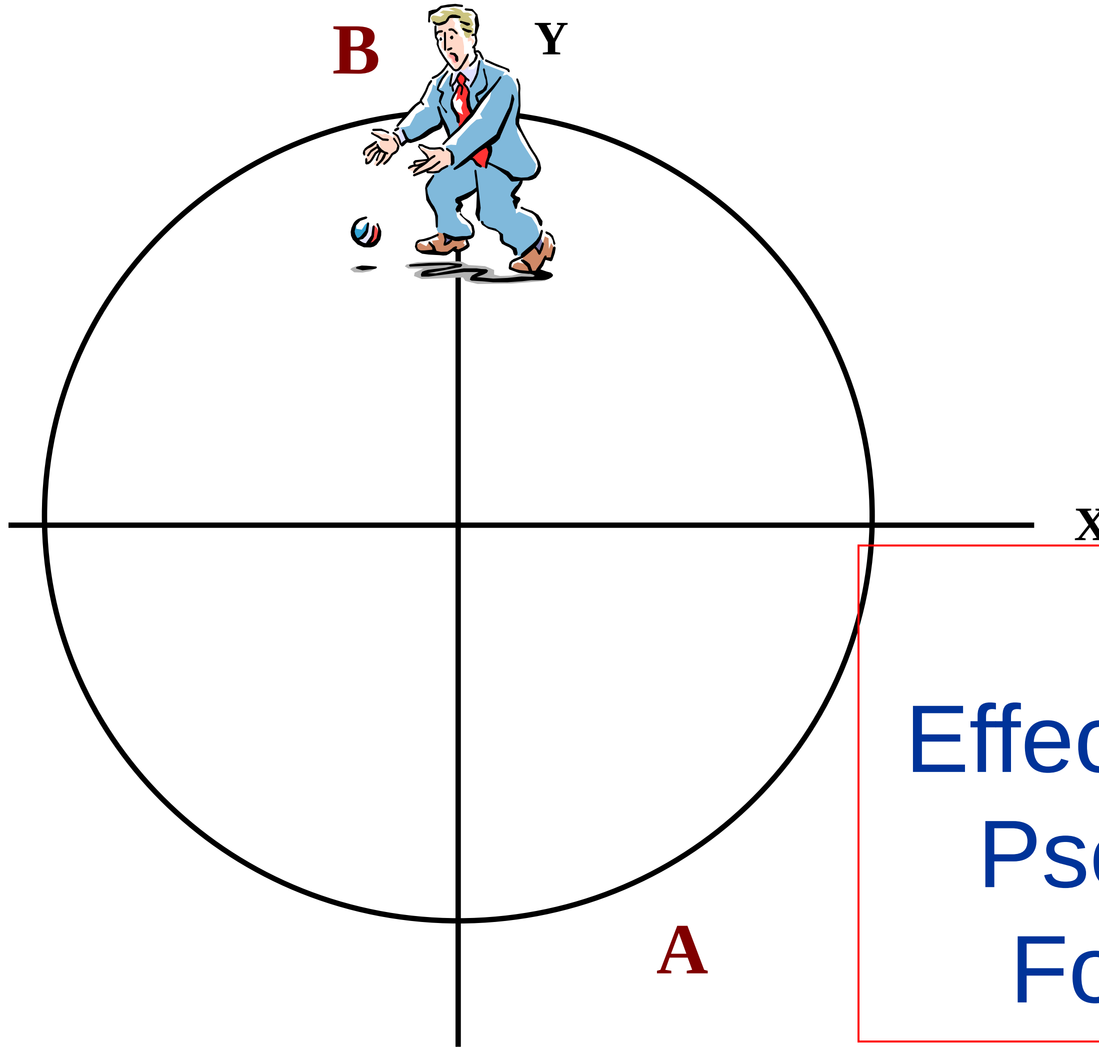
2 Astronauts in a Space Ship

Rotating

R = 10m

ω = 0.15 rad/sec

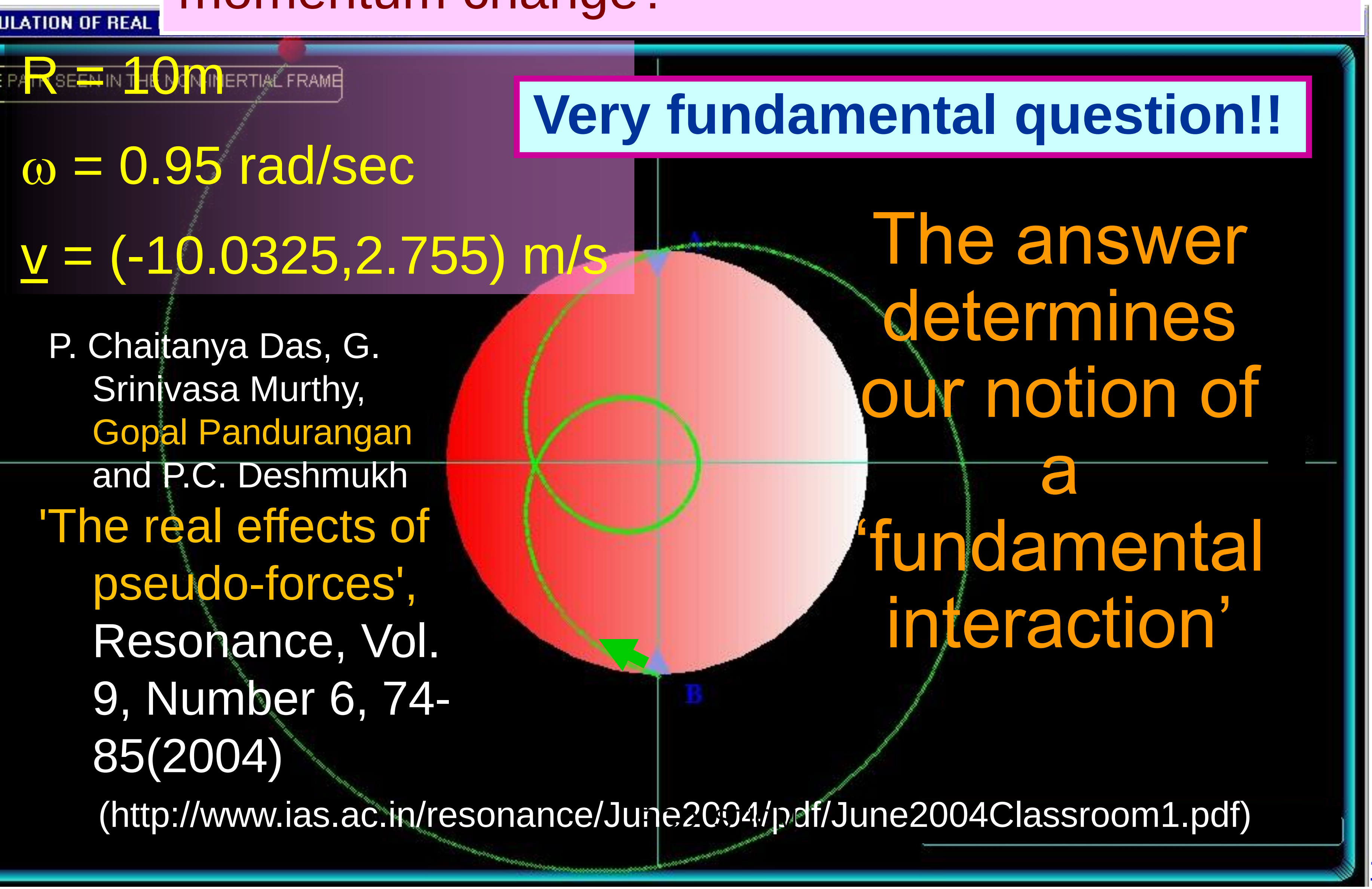
$\underline{v}$ = (0,5) m/sec



Real Effects of Pseudo Forces

Please read the paper at the following internet weblink:
http://www.physics.iitm.ac.in/~labs/amp/homepage/dasandmurthy.htm

Force = Rate of change of momentum
What interaction is making the instantaneous momentum change?



Very fundamental question!!

R = 10m

ω = 0.95 rad/sec

$\underline{v}$ = (-10.0325,2.755) m/s

The answer determines our notion of a ‘fundamental interaction’

P. Chaitanya Das, G. Srinivasa Murthy, Gopal Pandurangan and P.C. Deshmukh 'The real effects of pseudo-forces', Resonance, Vol. 9, Number 6, 74-85(2004)

(http://www.ias.ac.in/resonance/June2004/pdf/June2004Classroom1.pdf)

What will be the effect on rockets?

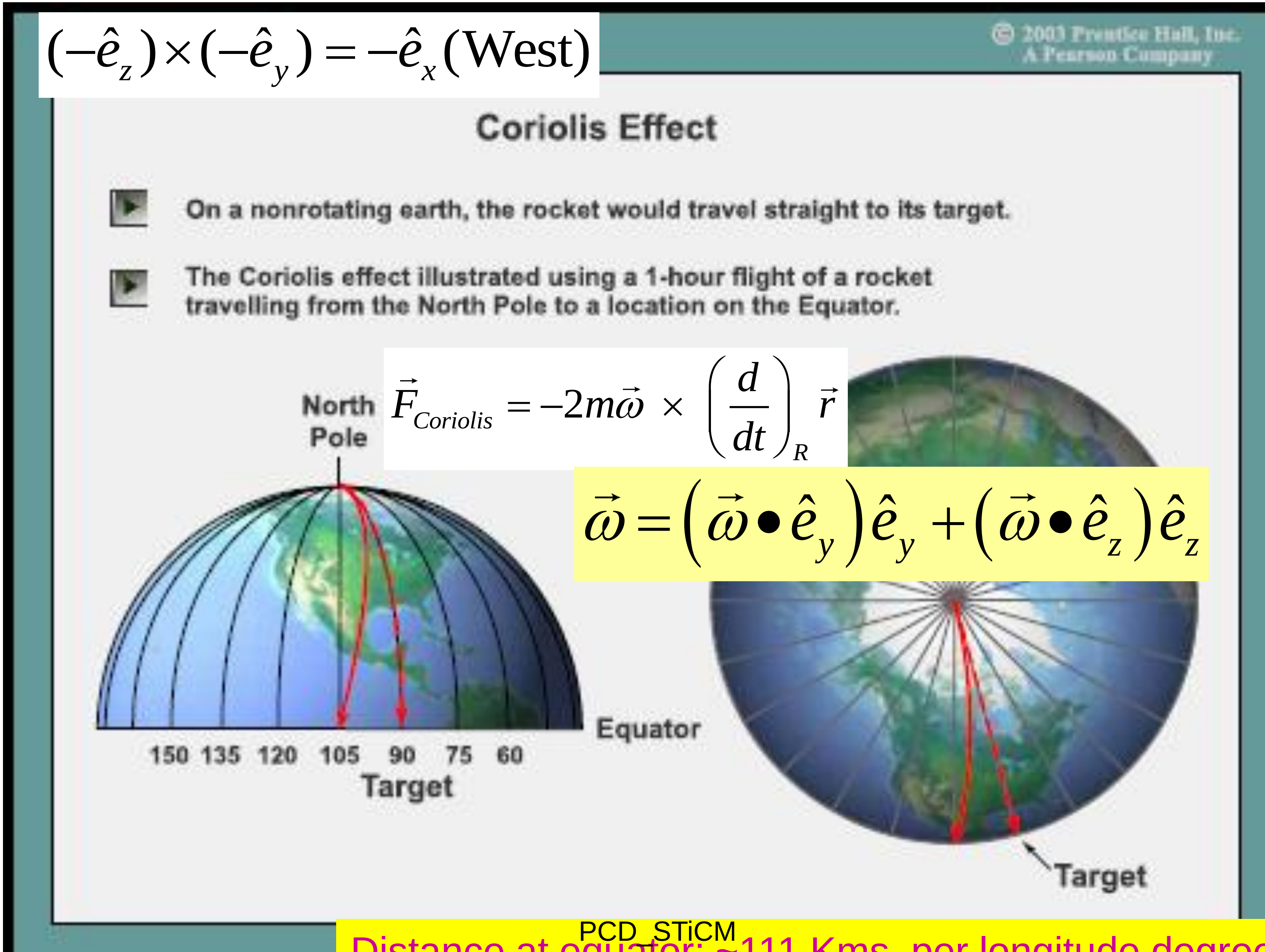
'Missile Woman of India'
Dr. Tessy Thomas

On ICBMs?

17th May, 2010 AGNI II

http://daphne.palomar.edu/pdeen/Animations/34_Coriolis.swf

$$(-\hat{e}_z)\times(-\hat{e}_y) = -\hat{e}_x(\text{West})$$



$$\vec{F}_{Coriolis} = -2m\vec{\omega}\times\left(\frac{d}{dt}\right)_R \vec{r}$$

$$\vec{\omega} = \left(\vec{\omega}\bullet\hat{e}_y\right)\hat{e}_y + \left(\vec{\omega}\bullet\hat{e}_z\right)\hat{e}_z$$

Distance at equator: ~111 Kms. per longitude degree

We will take a Break…

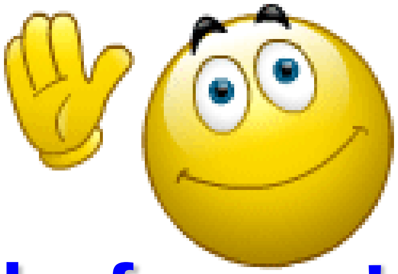
…… Any questions ?

pcd@physics.iitm.ac.in

Next L18 : Coriolis Deflection

Foucault Pendulum

Real Effects of Pseudo-forces!



$$\vec{F}_R = \vec{F}_I - \vec{F}_{\dot{\omega}} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r} - \mathrm{m}\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

STiCM

Select / Special Topics in Classical Mechanics

P. C. Deshmukh

Department of Physics
Indian Institute of Technology Madras
Chennai 600036

pcd@physics.iitm.ac.in

STiCM Lecture 18: Unit 5

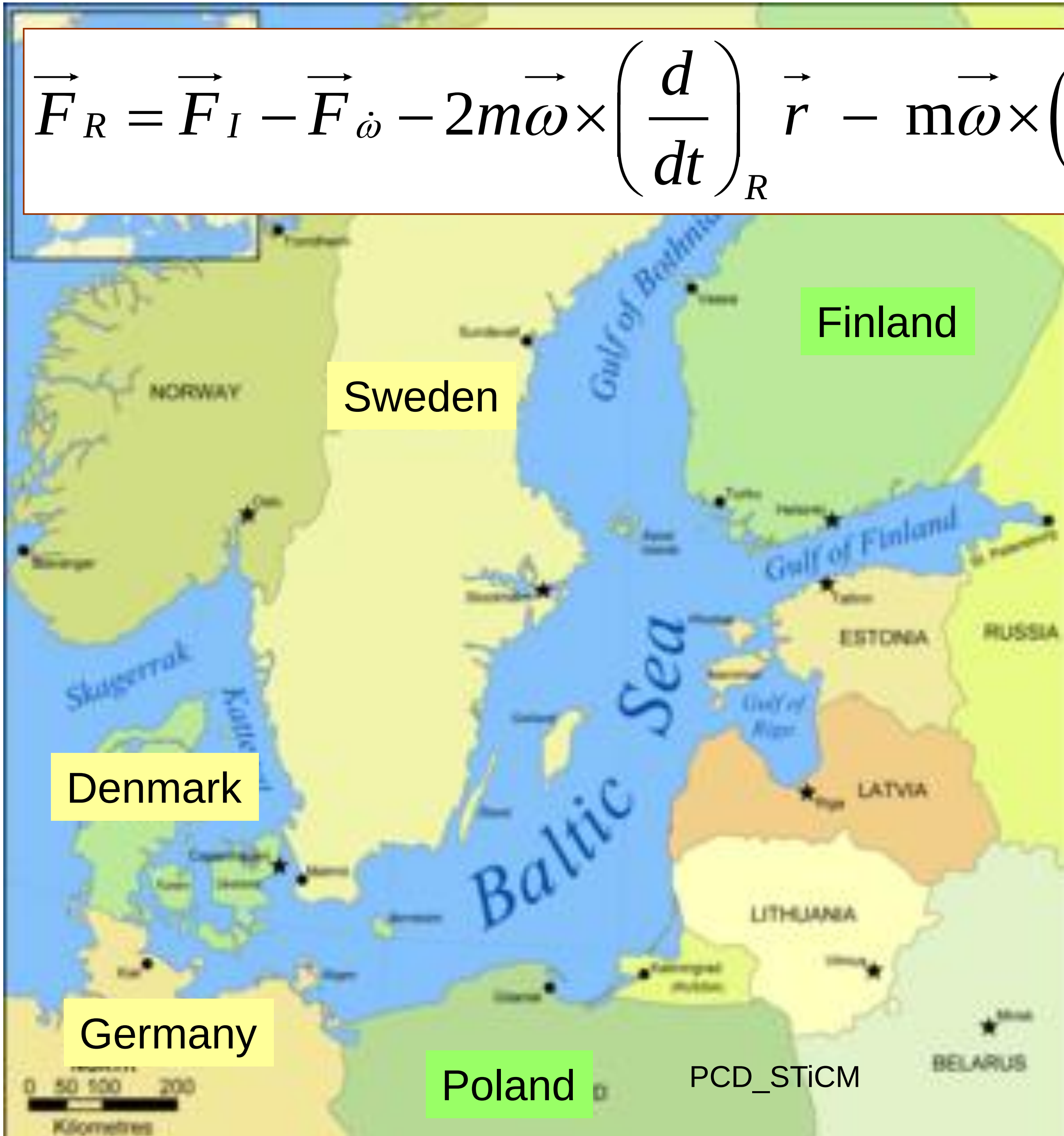
‘EFFECT’, when there isn’t a cause!

‘CAUSE’, when there really isn’t one!

Real Effects of Pseudo-forces!

Foucault Pendulum

$$\vec{F}_R = \vec{F}_I - \vec{F}_{\dot{\omega}} - 2m\vec{\omega}\times\left(\frac{d}{dt}\right)_R \vec{r} - \mathrm{m}\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$$



Physical Oceanography of the Baltic Sea

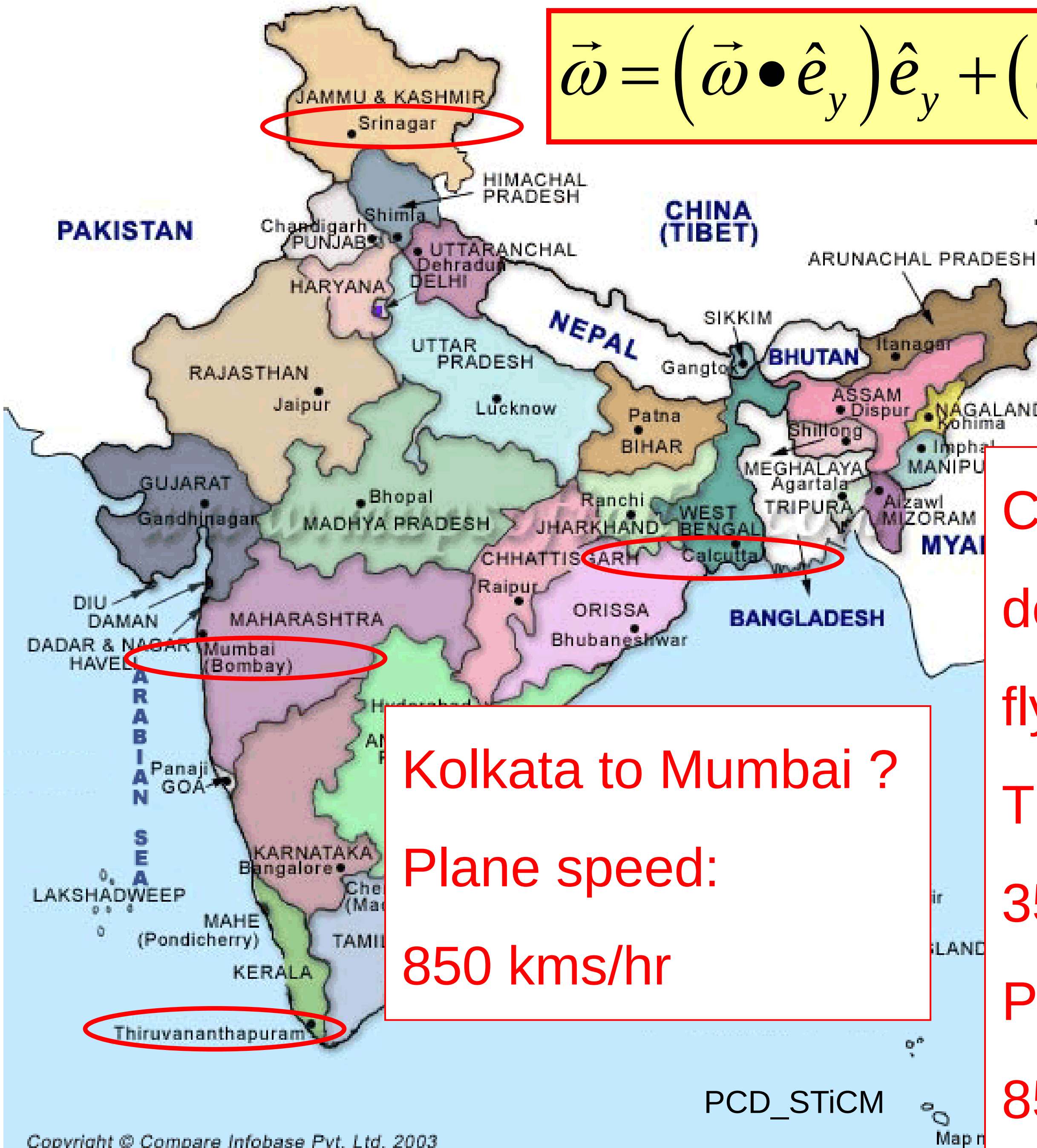
Matti Leppäranta, Kai Myrberg

Springer-Praxis (2009)

Circulating current in the Baltic sea (1969?)

$$\vec{\omega} = \left(\vec{\omega} \bullet \hat{e}_y\right)\hat{e}_y + \left(\vec{\omega} \bullet \hat{e}_z\right)\hat{e}_z$$

$$-2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$



Kolkata to Mumbai ?

Plane speed:

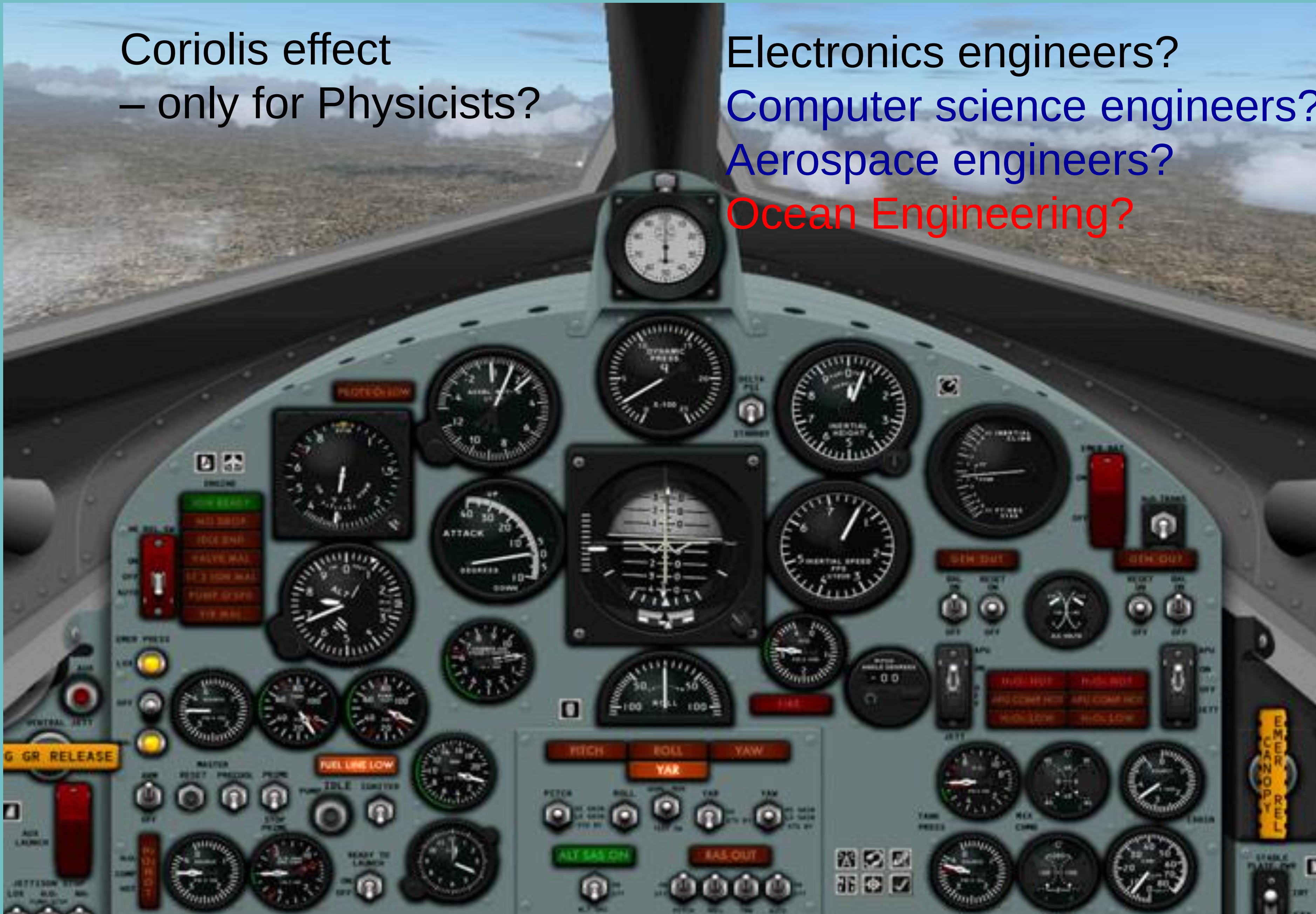
850 kms/hr

Calculate Coriolis deflection of a plane flying from Srinagar to Thiruvananthapuram ?

3550 kms

Plane speed:

850 kms/hr



Coriolis effect
– only for Physicists?

Electronics engineers?
Computer science engineers?
Aerospace engineers?
Ocean Engineering?

Any navigation system on earth….. GPS in cellphones!

Use a Cartesian coordinate system with reference to a point on the earth's surface.

Choose $\{\hat{e}_x, \hat{e}_y, \hat{e}_z\}$ such that $\hat{e}_z$ is along the local 'up/vertical' direction (which is **not** along $\hat{e}_r$ due to the centrifugal term).

$$\textit{However, } \hat{e}_z \textit{ is very nearly the same as } \hat{e}_r$$

Choose $\hat{e}_y$ such that it is orthogonal to $\hat{e}_z$, and points toward the North-pole seen from the point on the earth's surface under consideration.

Finally, choose $\hat{e}_x = \hat{e}_y \times \hat{e}_z$, which will give us the direction of the local 'East' at that point.

$$\vec{F}_{Coriolis} = -2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

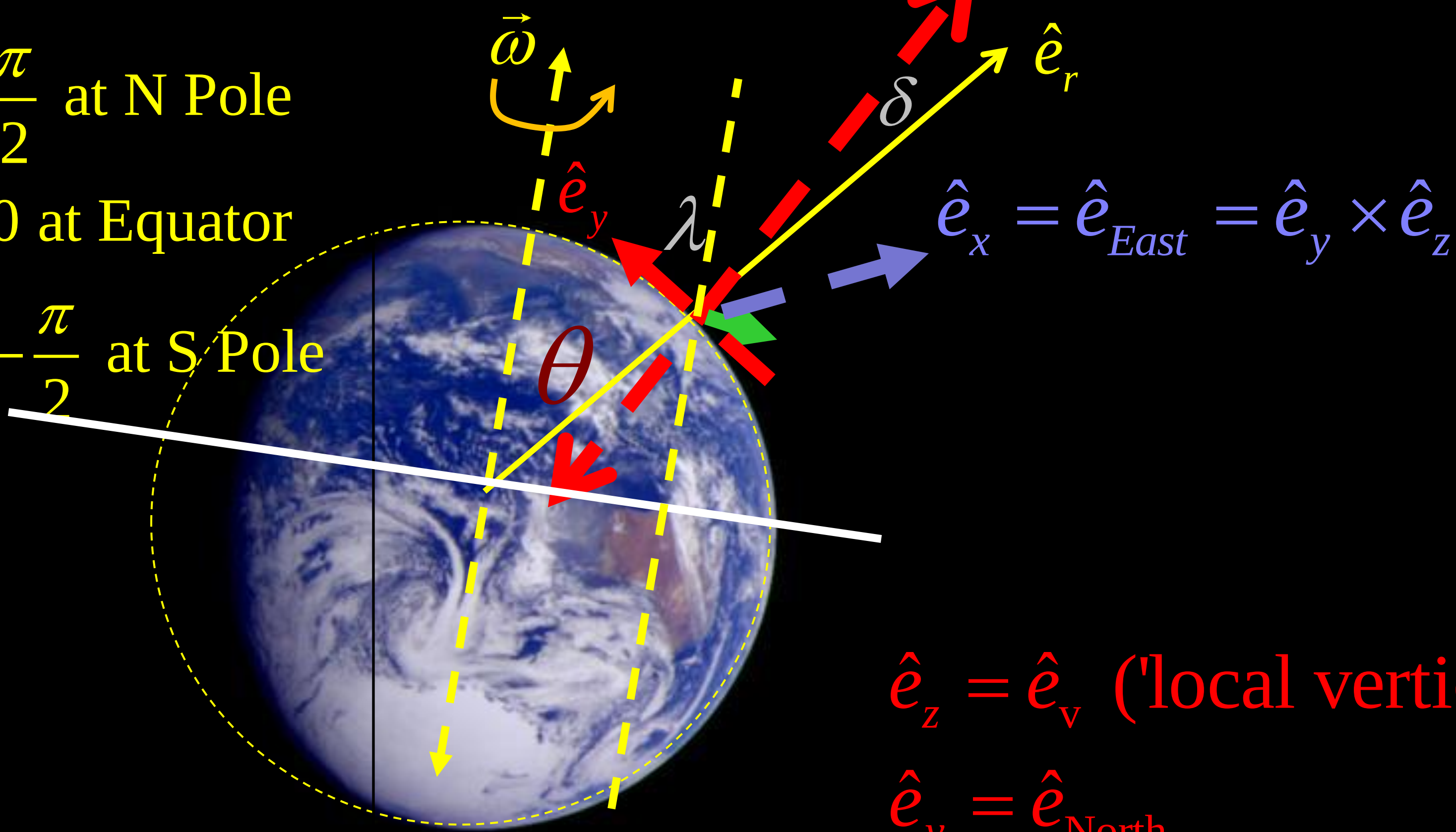
$$\boxed{\vec{\omega} = \left(\vec{\omega}\bullet\hat{e}_y\right)\hat{e}_y + \left(\vec{\omega}\bullet\hat{e}_z\right)\hat{e}_z}$$

$$\lambda = \measuredangle\left(\vec{\omega},\hat{e}_{North}\right) = \measuredangle\left(\vec{\omega},\hat{e}_y\right)$$

$\lambda = \frac{\pi}{2}$ at N Pole

$\lambda = 0$ at Equator

$\lambda = -\frac{\pi}{2}$ at S Pole



$\hat{e}_z = \hat{e}_v$ ('local vertical')

$\hat{e}_y = \hat{e}_{North}$

$\hat{e}_x = \hat{e}_{East} = \hat{e}_y \times \hat{e}_z$

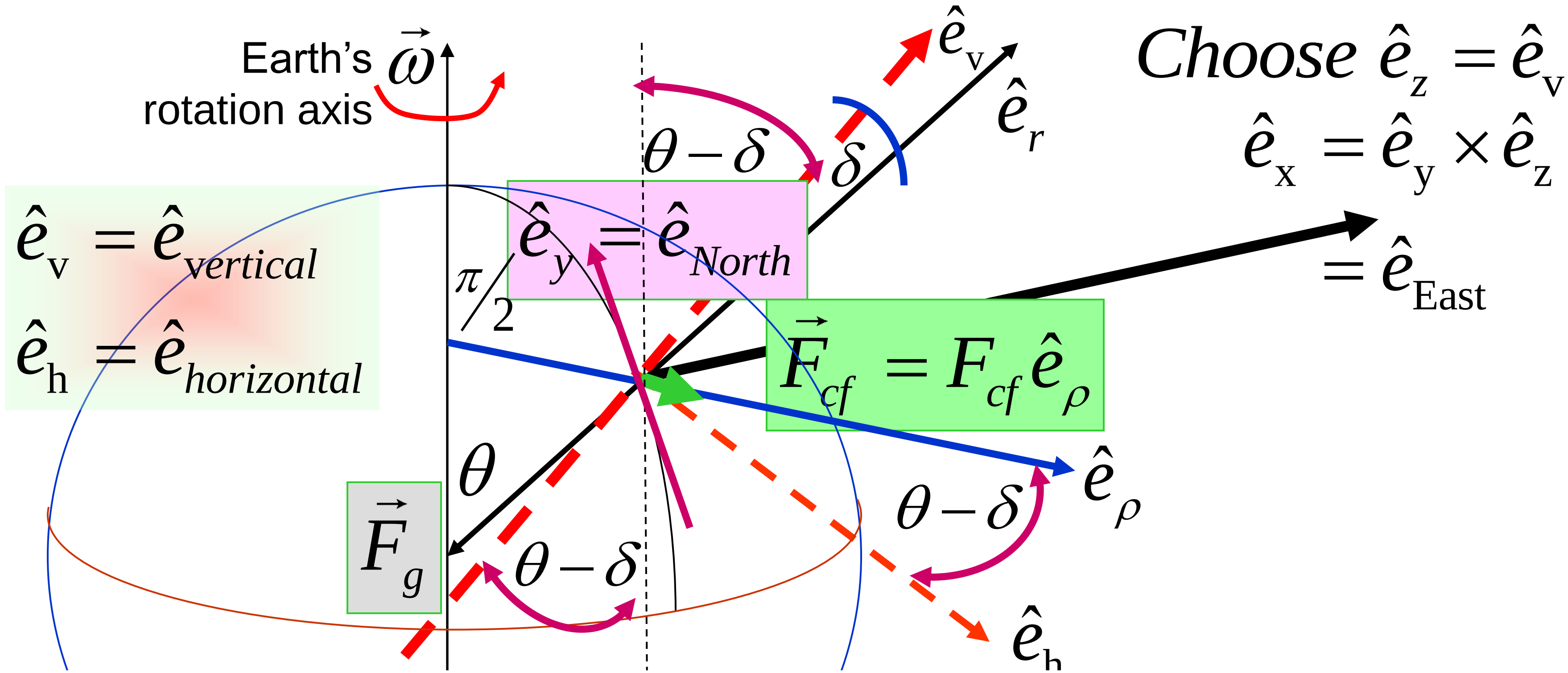
Caution! We use a mix of three coordinate systems!

1. A cartesian coordinate system as defined in the previous slide.

2. A spherical polar coordinate system whose 'polar' angle is defined with respect to the axis of the earth's rotation rather than with respect to the cartesian z-axis which is oriented along the local 'vertical'.

3. Cylindrical polar coordinate system whose radial unit vector is along the radial outward direction with reference to the earth's <u>axis</u> of rotation.

Just follow the 6 steps indicated in the next slide, exactly in the order given!



The 'vertical' is *<u>not</u>* along the radial line, nor along the axis of earth's rotation!

$$\vec{\omega} = \left(\vec{\omega}\bullet\hat{e}_y\right)\hat{e}_y + \left(\vec{\omega}\bullet\hat{e}_z\right)\hat{e}_z$$

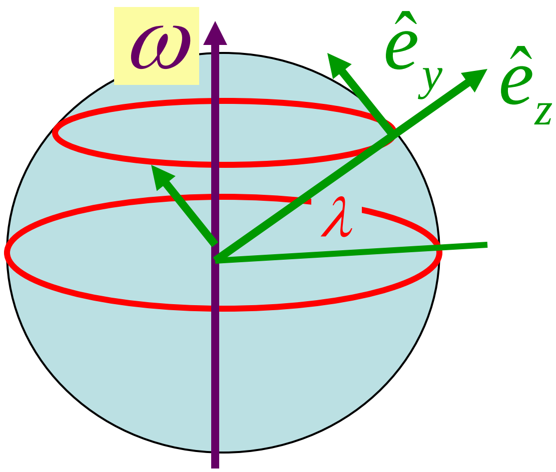
$$\vec{F}_R = \vec{F}_I - \cancel{\vec{F}_{\dot{\omega}}} - 2m\vec{\omega}\times\left(\frac{d}{dt}\right)_R \vec{r} \text{ - } m\cancel{\vec{\omega}\times}\left(\vec{\omega}\times\vec{r}\right)$$

$$\boxed{\vec{\omega} = \left(\vec{\omega}\bullet\hat{e}_y\right)\hat{e}_y + \left(\vec{\omega}\bullet\hat{e}_z\right)\hat{e}_z}$$

$$\left(\frac{d}{dt}\right)_R \vec{r} = \left[\mathrm{v}_x\hat{e}_x + \mathrm{v}_y\hat{e}_y + \mathrm{v}_z\hat{e}_z\right]$$

Velocity of the object in ROTATING FRAME

$$\boxed{\lambda : latitude}$$

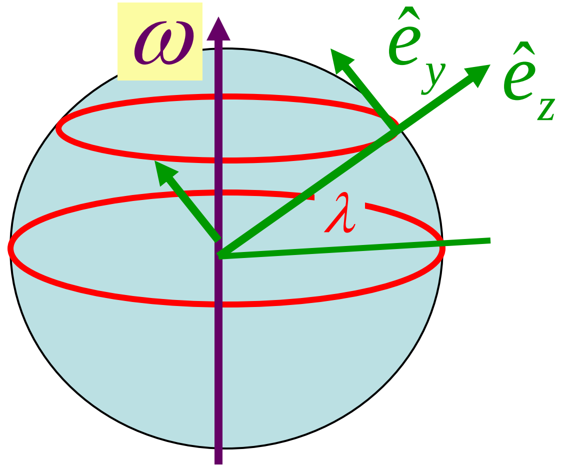


$$\vec{F}_{Coriolis} = -2m\left[\left(\vec{\omega}\bullet\hat{e}_y\right)\hat{e}_y + \left(\vec{\omega}\bullet\hat{e}_z\right)\hat{e}_z\right]\times\left[\mathrm{v}_x\hat{e}_x + \mathrm{v}_y\hat{e}_y + \mathrm{v}_z\hat{e}_z\right]$$

$$\overrightarrow{F}_{Coriolis} = -2m\omega\left[\cos\lambda\hat{e}_y + \sin\lambda\hat{e}_z\right]\times\left[\mathrm{v}_x\hat{e}_x + \mathrm{v}_y\hat{e}_y + \mathrm{v}_z\hat{e}_z\right]$$

$$m\vec{a}_{Coriolis} = -2m\omega\left[\left(\cos\lambda\mathrm{v}_z - \sin\lambda\mathrm{v}_y\right)\hat{e}_x + \sin\lambda\mathrm{v}_x\hat{e}_y + (-\cos\lambda\mathrm{v}_x)\hat{e}_z\right]$$

$\lambda : latitude$



Pendulum at the N pole

The observer is at the N pole.

How will the plane of oscillation of the pendulum look to an observer at the N pole?

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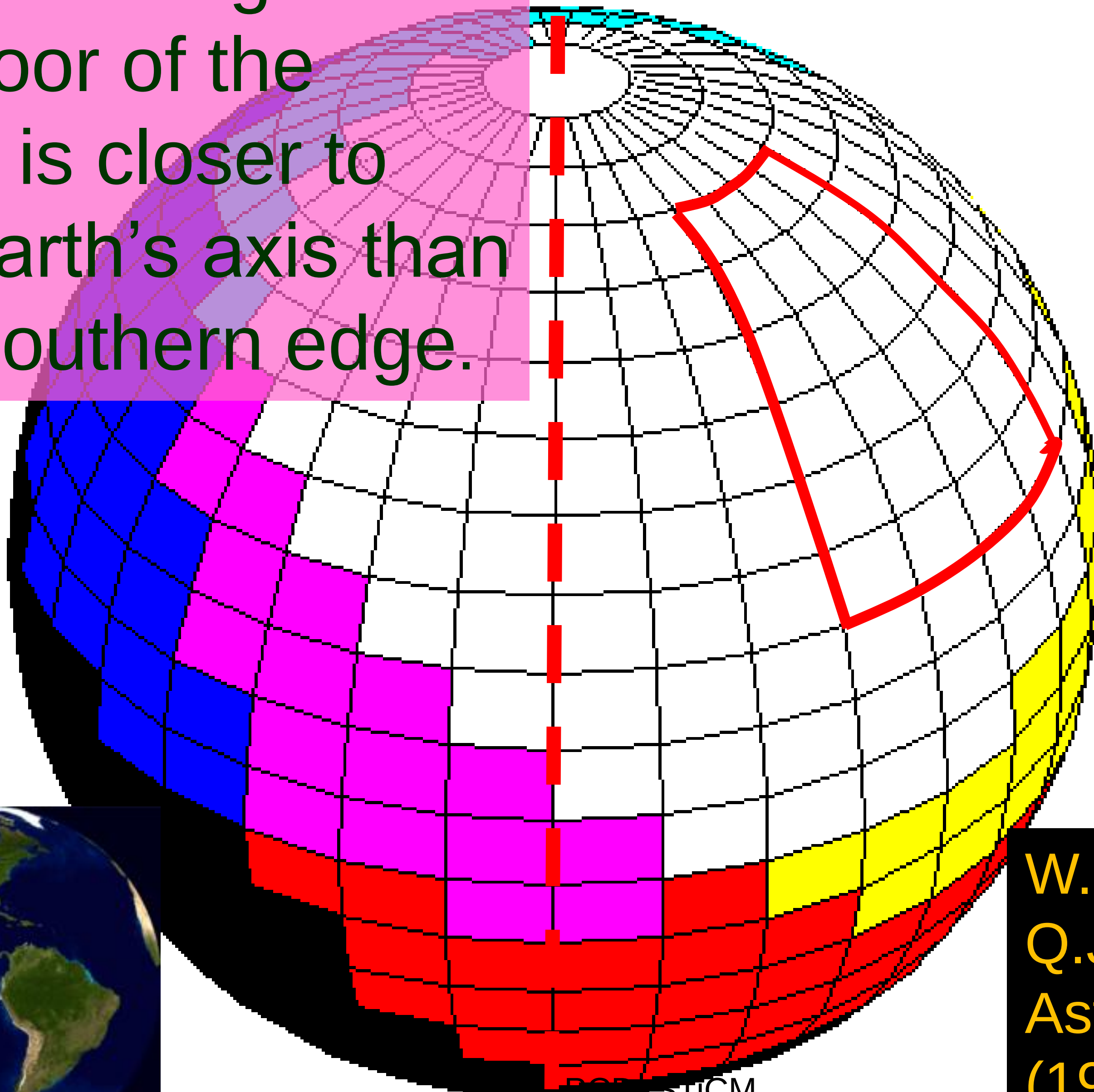
Pendulum at the S pole

How will the plane of oscillation of the pendulum look to an observer at the S pole?

What if the pendulum is at an intermediate latitude?

How will the plane of oscillation of the pendulum look to an observer at that latitude?

Northern edge of the floor of the room is closer to the earth’s axis than the Southern edge.



Northern edge of the floor moves eastward slower than the Southern edge.

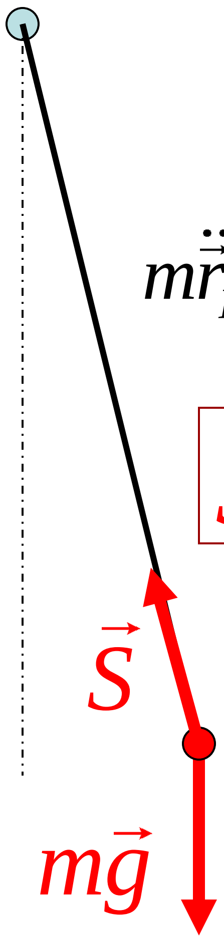
W.B.Somerville Q.Jl. Royal Astronomical Soc. (1972) <u>13</u> 40-62 http://www.youtube.com/watch?v=nB2SXLYwKkM

video compilation of a Foucault pendulum in action

at the Houston Museum of Natural Science.

Foucault Pendulum

$$\boxed{\vec{\omega}=\left(\vec{\omega}\bullet\hat{e}_y\right)\hat{e}_y+\left(\vec{\omega}\bullet\hat{e}_z\right)\hat{e}_z}$$



$$\vec{F}_R=\vec{F}_I-\vec{F}_{\dot{\omega}}-2m\vec{\omega}\times\left(\frac{d}{dt}\right)_R\vec{r}\ \text{-}\ \mathrm{m}\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$$

$$m\ddot{\vec{r}}_R=m\vec{g}+\vec{S}-\cancel{\vec{F}_{\dot{\omega}}}-2m\vec{\omega}\times\left(\frac{d}{dt}\right)_R\vec{r}\ \text{-}\ \cancel{\mathrm{m}\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)}$$

$$\boxed{\vec{S}=S\hat{u}}$$

$$\vec{S}=\hat{e}_x(\hat{e}_x\bullet\vec{S})+\hat{e}_y(\hat{e}_y\bullet\vec{S})+\hat{e}_z(\hat{e}_z\bullet\vec{S})$$

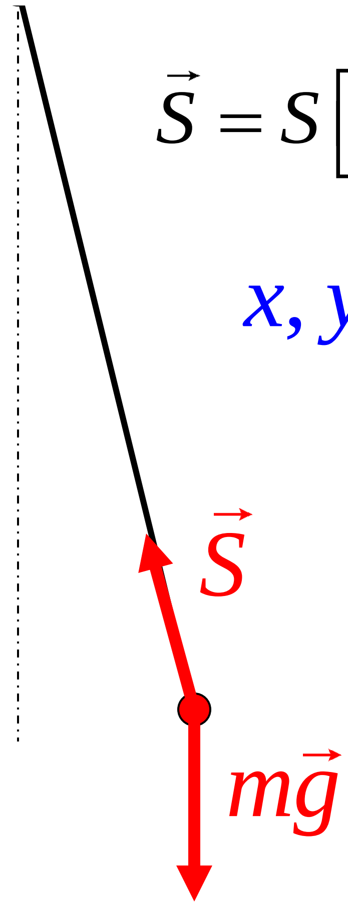
$$\vec{S}=S\left[\hat{e}_x(\hat{e}_x\bullet\hat{u})+\hat{e}_y(\hat{e}_y\bullet\hat{u})+\hat{e}_z(\hat{e}_z\bullet\hat{u})\right]$$

$$\vec{S}=S\left[\hat{e}_x\cos\alpha+\hat{e}_y\cos\beta+\hat{e}_z\cos\gamma\right]$$

Foucault Pendulum

$$m\ddot{\vec{r}}_R = m\vec{g} + \vec{S} - 2m\vec{\omega}\times\left(\frac{d}{dt}\right)_R \vec{r}$$

$$m\vec{a}_{Coriolis} = -2m\omega\left[\left(\cos\cancel{\lambda \mathrm{v}_z} - \sin\lambda \mathrm{v}_y\right)\hat{e}_x + \sin\lambda \mathrm{v}_x \hat{e}_y + (-\cos\lambda \mathrm{v}_x)\hat{e}_z\right]$$



$$\vec{S} = S\left[\hat{e}_x\cos\alpha + \hat{e}_y\cos\beta + \hat{e}_z\cos\gamma\right]$$

$\boxed{\text{neglect } \dot{z}}$

x, y motion:

$$m\ddot{x} = S\cos\alpha - 2m\omega\left(\cos\cancel{\lambda\dot{z}} - \sin\lambda\dot{y}\right)$$

$$m\ddot{y} = S\cos\beta - 2m\omega\sin\lambda\dot{x}$$

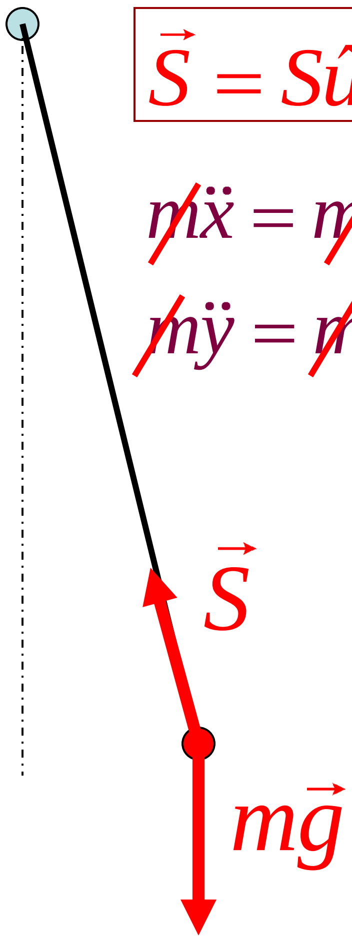
$$S \approx mg \Rightarrow$$

$$m\ddot{x} = mg\cos\alpha + 2m\omega\sin\lambda\dot{y}$$

$$m\ddot{y} = mg\cos\beta - 2m\omega\sin\lambda\dot{x}$$

Foucault Pendulum

$$m\ddot{\vec{r}}_R = m\vec{g} + \vec{S} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$



$\boxed{\vec{S} = S\hat{u}}$ $\boxed{neglect\ \dot{z}}$ $\boxed{S \approx mg}$

$$\cancel{m}\ddot{x} = \cancel{m}g\cos\alpha + 2\cancel{m}\omega\sin\lambda\dot{y}$$

$$\cancel{m}\ddot{y} = \cancel{m}g\cos\beta - 2\cancel{m}\omega\sin\lambda\dot{x}$$

$$\ddot{x} = g\cos\alpha + 2\omega\sin\lambda\dot{y}$$

$$\ddot{y} = g\cos\beta - 2\omega\sin\lambda\dot{x}$$

$$\boxed{\Omega = \omega\sin\lambda}$$

$$\ddot{x} = g\cos\alpha + 2\,\Omega\,\dot{y}$$

$$\ddot{y} = g\cos\beta - 2\,\Omega\,\dot{x}$$

$$\ddot{x} = g\cos\alpha + 2\Omega\,\dot{y}$$
$$\ddot{y} = g\cos\beta - 2\Omega\,\dot{x}$$

Coupled differential equations

Solve by transforming to new coordinates x’,y’

such that

$$\boxed{\begin{aligned} x &= x'\cos(\Omega t) + y'\sin(\Omega t) \\ y &= -x'\sin(\Omega t) + y'\cos(\Omega t) \end{aligned}}$$

$$\dot{x} = -\Omega\left[x'\sin(\Omega t) - y'\cos(\Omega t)\right]$$
$$\dot{y} = -\Omega\left[x'\cos(\Omega t) + y'\sin(\Omega t)\right]$$

$$\ddot{x} = g\cos\alpha - 2\Omega^2\left[x'\cos(\Omega t) + y'\sin(\Omega t)\right]$$
$$\ddot{y} = g\cos\beta + 2\Omega^2\left[x'\sin(\Omega t) - y'\cos(\Omega t)\right]$$

$$\ddot{x} = g\cos\alpha - 2\Omega^2\left[x'\cos(\Omega t) + y'\sin(\Omega t)\right]$$

$$\ddot{y} = g\cos\beta + 2\Omega^2\left[x'\sin(\Omega t) - y'\cos(\Omega t)\right]$$

$$\boxed{\begin{aligned}\ddot{x}\cos(\Omega t) &= g\cos\alpha\cos(\Omega t) - 2\Omega^2\left[x'\cos^2(\Omega t) + y'\sin(\Omega t)\cos(\Omega t)\right] \\ \ddot{y}\sin(\Omega t) &= g\cos\beta\sin(\Omega t) + 2\Omega^2\left[x'\sin^2(\Omega t) - y'\cos(\Omega t)\sin(\Omega t)\right]\end{aligned}}$$

$$\boxed{\begin{aligned}\ddot{x}\cos(\Omega t) + \ddot{y}\sin(\Omega t) = g\cos\alpha\cos(\Omega t) - 2\Omega^2\left[x'\cos^2(\Omega t) + y'\sin(\Omega t)\cos(\Omega t)\right] \\ + g\cos\beta\sin(\Omega t) + 2\Omega^2\left[x'\sin^2(\Omega t) - y'\cos(\Omega t)\sin(\Omega t)\right]\end{aligned}}$$

$$\boxed{\begin{aligned}\ddot{x}\cos(\Omega t)+\ddot{y}\sin(\Omega t)&=g\cos\alpha\cos(\Omega t)-2\Omega^2\left[x'\cos^2(\Omega t)+y'\sin(\Omega t)\cos(\Omega t)\right]\\&\quad+g\cos\beta\sin(\Omega t)+2\Omega^2\left[x'\sin^2(\Omega t)-y'\cos(\Omega t)\sin(\Omega t)\right]\end{aligned}}$$

Dropping terms in Ω^2

$$\left(\ddot{x}-g\cos\alpha\right)\cos(\Omega t)+\left(\ddot{y}-g\cos\beta\right)\sin(\Omega t)=0$$

$$\left(\ddot{x}-g\cos\alpha\right)=0$$

$$\left(\ddot{y}-g\cos\beta\right)=0$$

$$\boxed{\Omega = \omega \sin \lambda}$$

$$m\ddot{\vec{r}}_R = m\vec{g} + \vec{S} - 2m\overrightarrow{\omega} \times \left(\frac{d}{dt} \right)_R \vec{r}$$

Direction cosines of $\vec{\mathrm{S}}$ are:

$$\cos \alpha = -\frac{x}{l}$$

$$\cos \beta = -\frac{y}{l}$$

$$\cos \gamma = \frac{z-l}{l}$$

$$\left(\ddot{x} - g \cos \alpha \right) = 0$$

$$\left(\ddot{y} - g \cos \beta \right) = 0$$

$$\left(\ddot{x} + g \frac{x}{l} \right) = 0$$

$$\left(\ddot{y} + g \frac{y}{l} \right) = 0$$

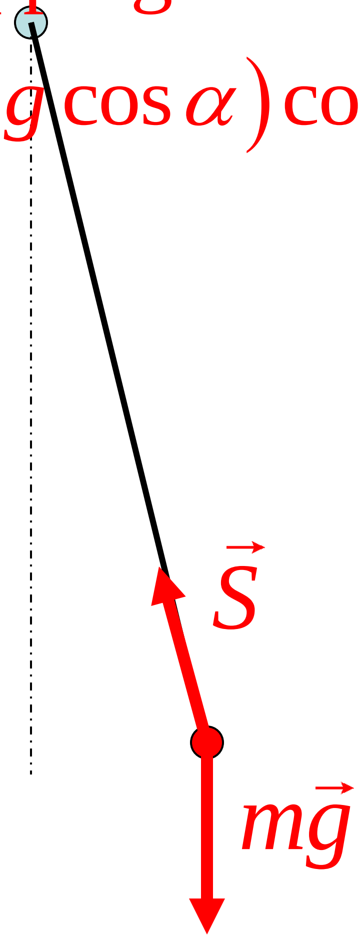
Two-dimensional linear harmonic oscillator

Foucault Pendulum

$$m\ddot{\vec{r}}_R = m\vec{g} + \vec{S} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

Dropping terms in Ω^2

$$\left(\ddot{x} - g\cos\alpha\right)\cos\left(\Omega t\right) + \left(\ddot{y} - g\cos\beta\right)\sin\left(\Omega t\right) = 0$$



$$\left(\ddot{x} + g\frac{x}{l}\right) = 0; \quad \left(\ddot{y} + g\frac{y}{l}\right) = 0$$

In the earth's rotating frame, the path is that of an ellipse.

Foucault Pendulum

$$m\ddot{\vec{r}}_R = m\vec{g} + \vec{S} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

Dropping terms in Ω^2

$$(\ddot{x} - g\cos\alpha)\cos(\Omega t) + (\ddot{y} - g\cos\beta)\sin(\Omega t) = 0$$

$$\left(\ddot{x} + g\frac{x}{l}\right) = 0; \quad \left(\ddot{y} + g\frac{y}{l}\right) = 0$$

The ellipse would precess at an angular speed $\Omega = \omega\sin\lambda$

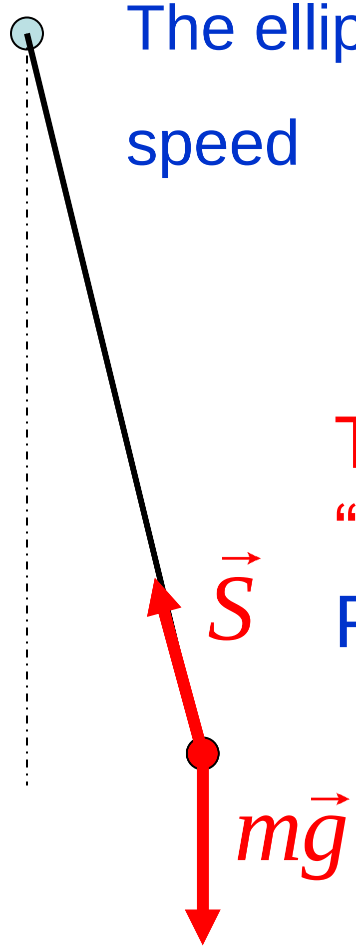
A number of approximations made!

Detailed analysis is rather involved!

The "plane" would in fact be a "curved surface".

Foucault Pendulum

$$m\ddot{\vec{r}}_R = m\vec{g} + \vec{S} - 2m\overrightarrow{\omega}\times\left(\frac{d}{dt}\right)_R\vec{r}$$



The ellipse would precess at an angular speed $\Omega = \omega\sin\lambda$

$\lambda : latitude$

Time Period for the rotation of the "plane" of oscillation of the Foucault Pendulum

$$T = \frac{1}{f} = \frac{2\pi}{\Omega} = \frac{2\pi}{\omega\sin\lambda}$$

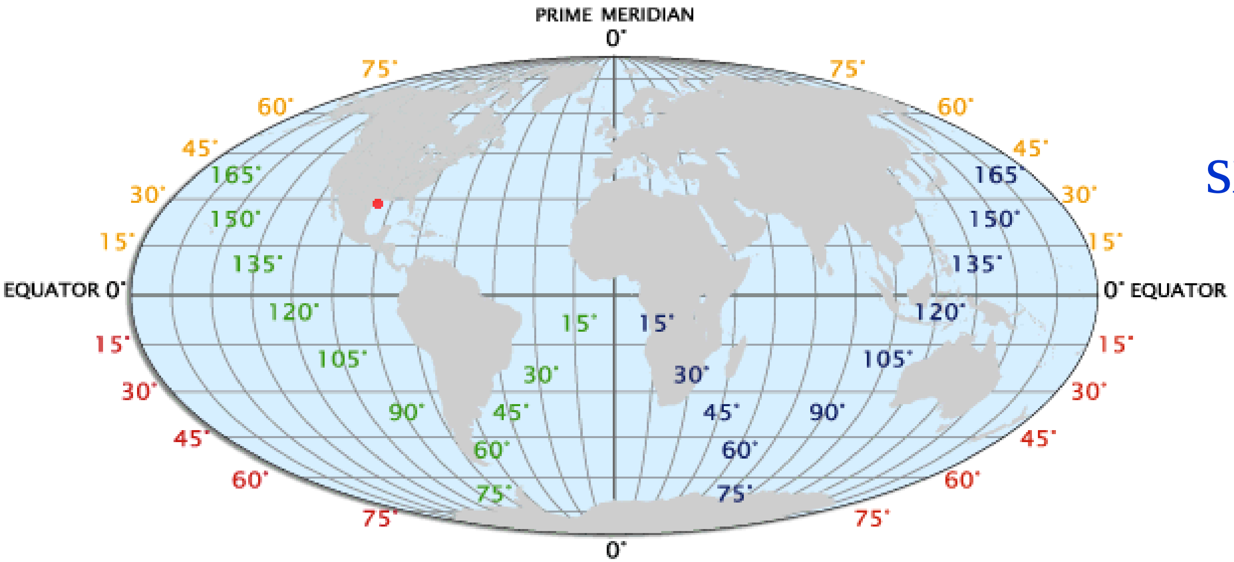
$$= \frac{2\pi}{2\pi\nu\sin\lambda} = \frac{24\text{ hours}}{\sin\lambda}$$

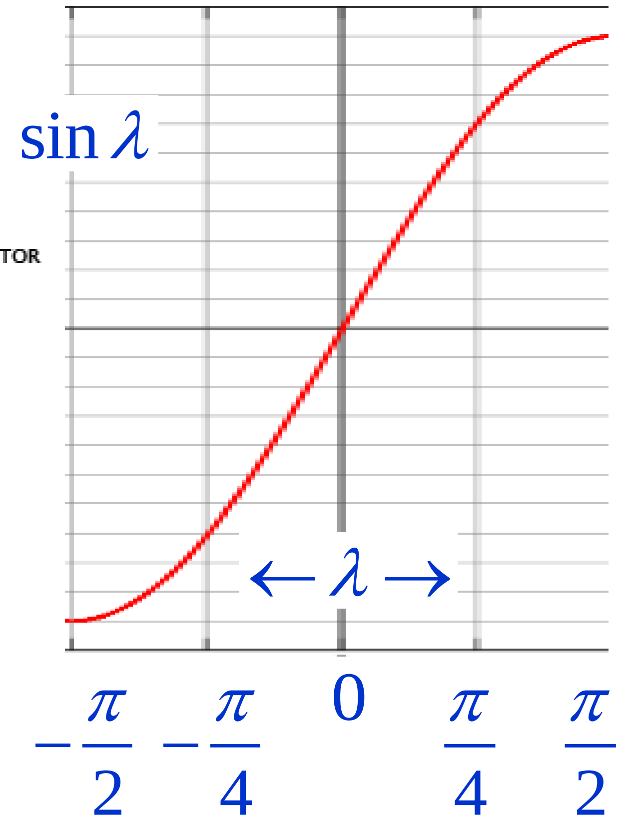
Famous Foucault Pendulum:

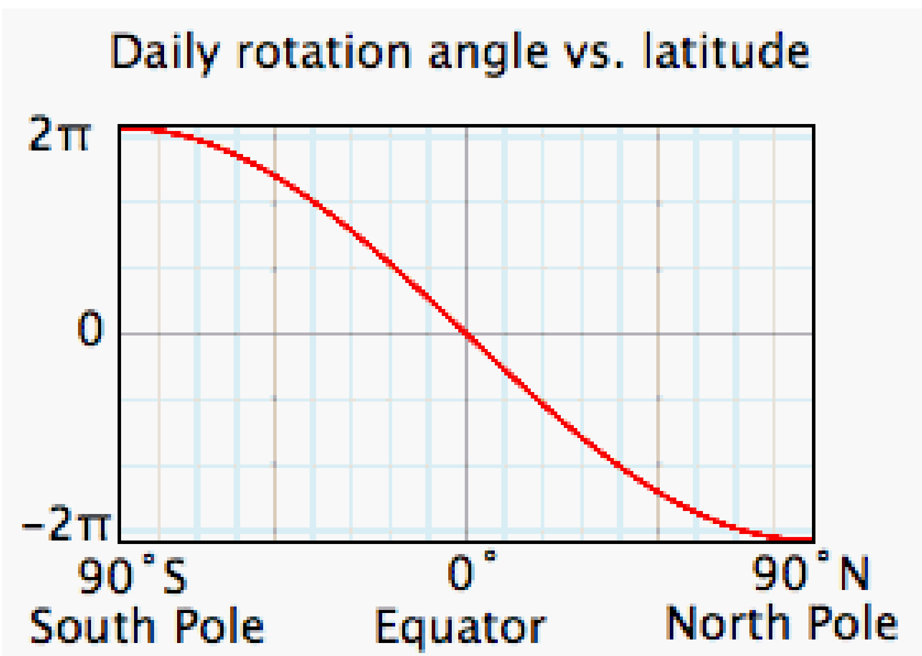
- at the Pantheon in Paris, France.

http://www.animations.physics.unsw.edu.au/jw/foucault_pendulum.html

http://www.youtube.com/watch?v=vVg5P6frHzY&feature=related







$$T = \frac{1}{f} = \frac{2\pi}{\Omega} = \frac{2\pi}{\omega \sin\lambda}$$

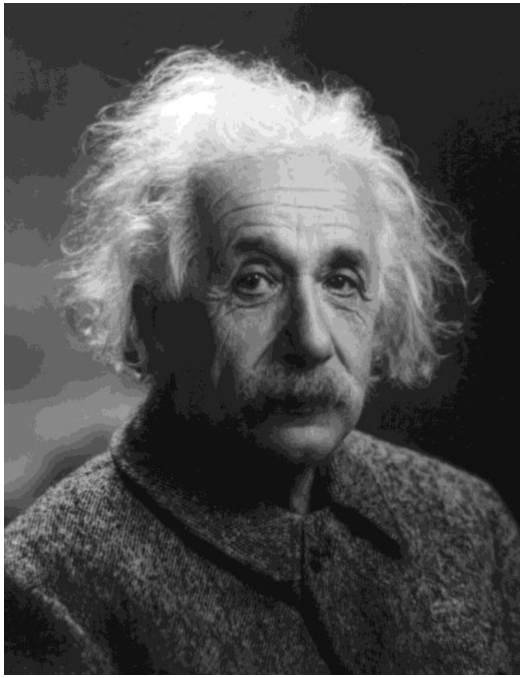
$$= \frac{2\pi}{2\pi\nu \sin\lambda} = \frac{24 \text{ hours}}{\sin\lambda}$$

Philosophical questions:

What is ‘force’ ? Mass/ Inertial frame? Gravity?

Sir Isaac Newton
1643 - 1727
From a portrait by
Enoch Seeman in 1726

Ernst Mach (1838–1916)



Albert Einstein
1879 – 1955

Newton: Gravity is the result of an attractive interaction between objects having mass.

Curvature of Space-Time,
Geometry / Dynamics of Matter / General Theory of Relativity

We will take a Break…

…… Any questions ?

pcd@physics.iitm.ac.in

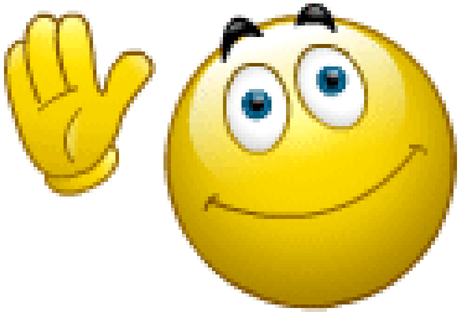
- In the next Unit, we shall consider

  Lorentz Transformations

  and Einstein's

  Special Theory of Relativity.

- *c: finite!*



Next, Unit 6: Special Theory of Relativity